

# A Post-Quantum Round-Optimal Oblivious PRF from Isogenies

Andrea Basso



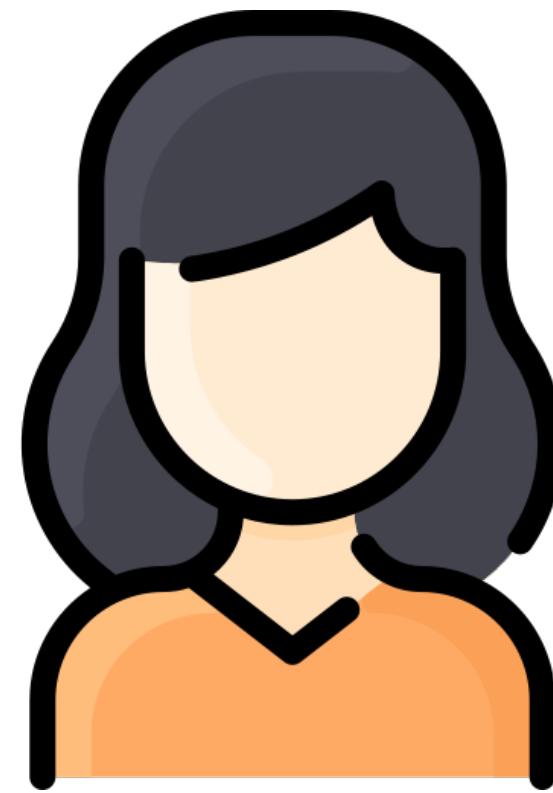
UNIVERSITY OF  
BIRMINGHAM



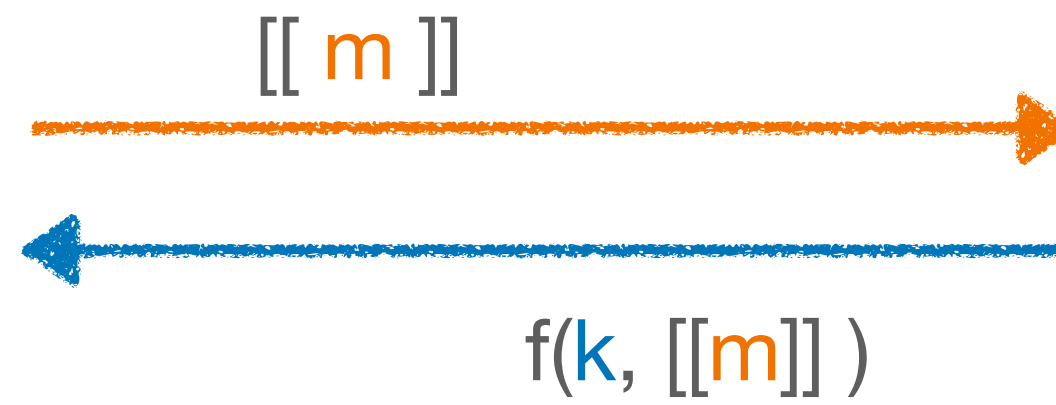
University of  
BRISTOL

16<sup>th</sup> August, 2023  
Selected Areas in Cryptography 2023

# Oblivious PRF

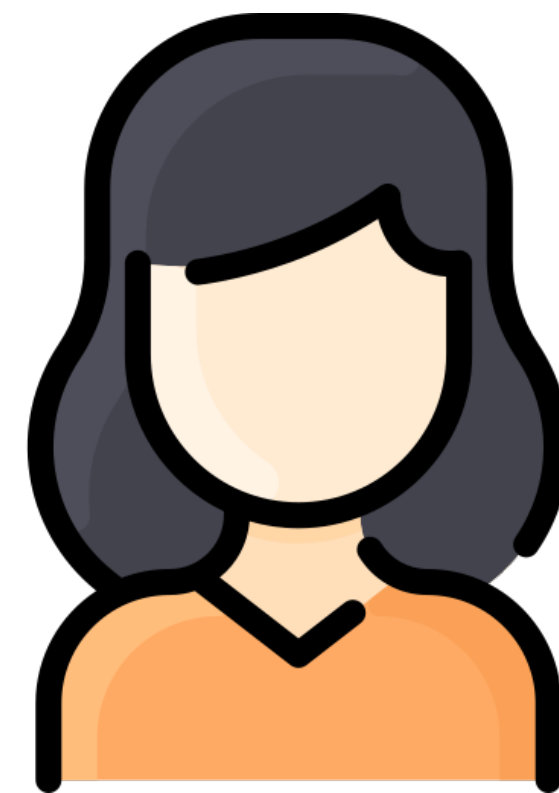


User



Server

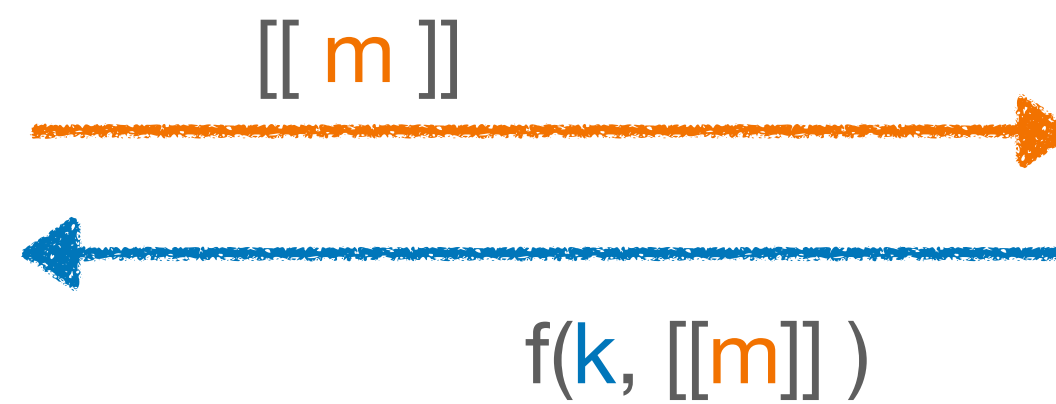
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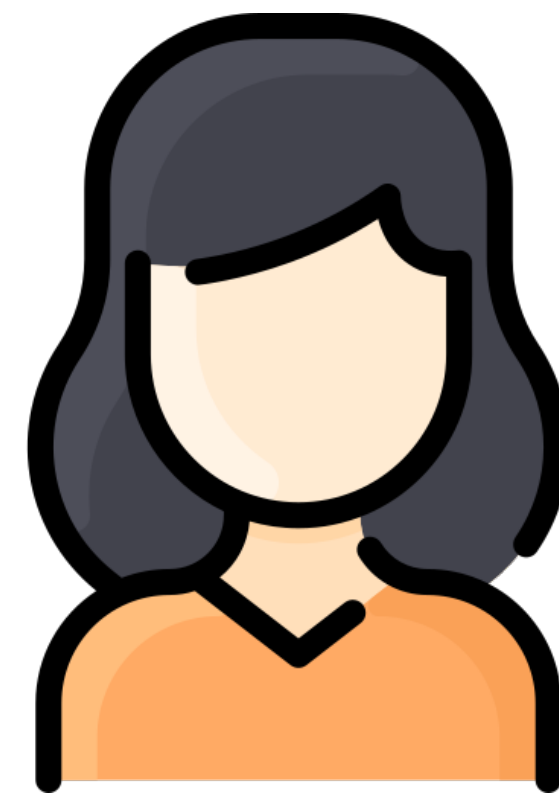


$F(k, m)$



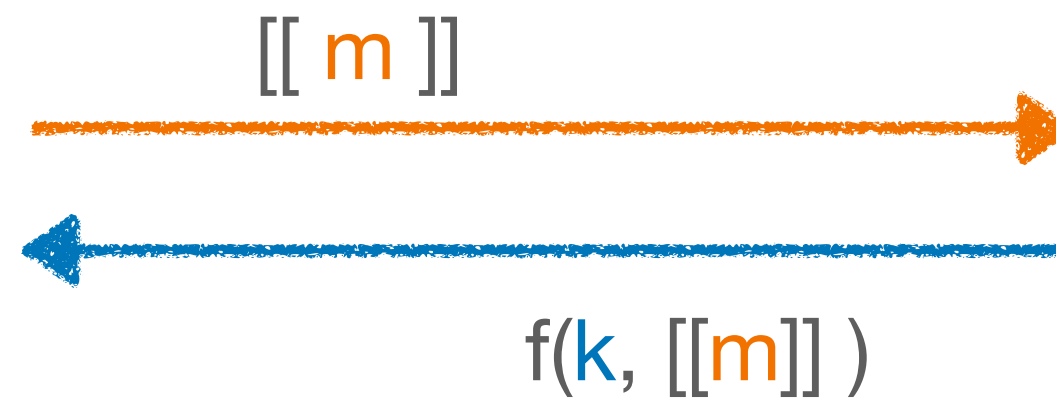
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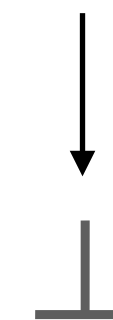


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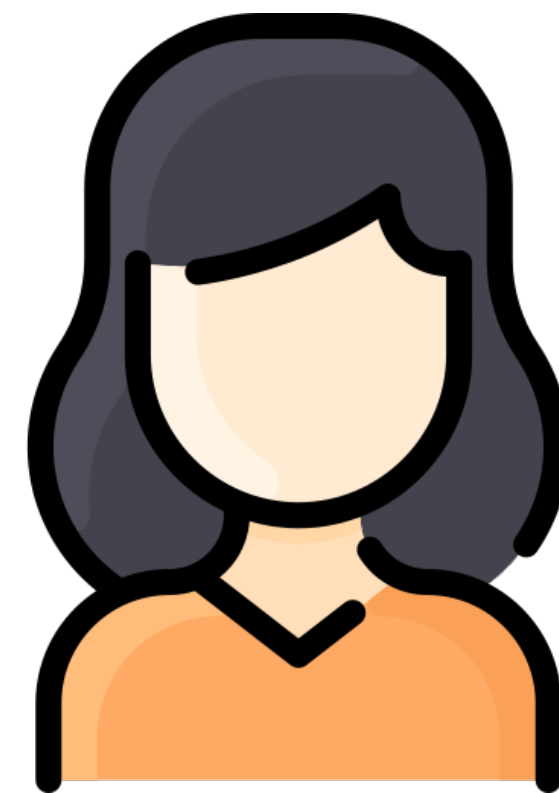
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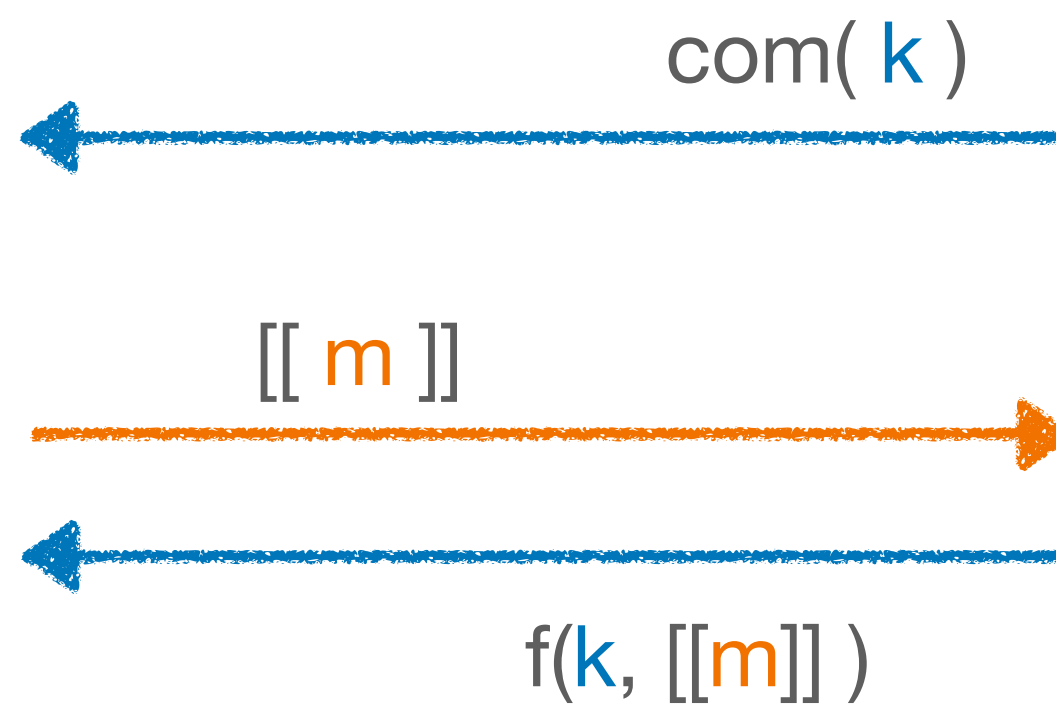


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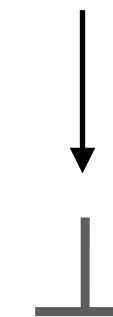


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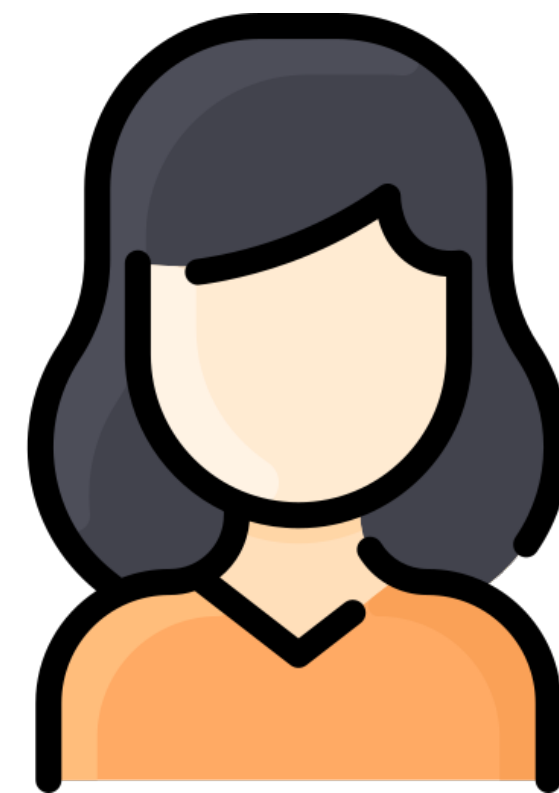
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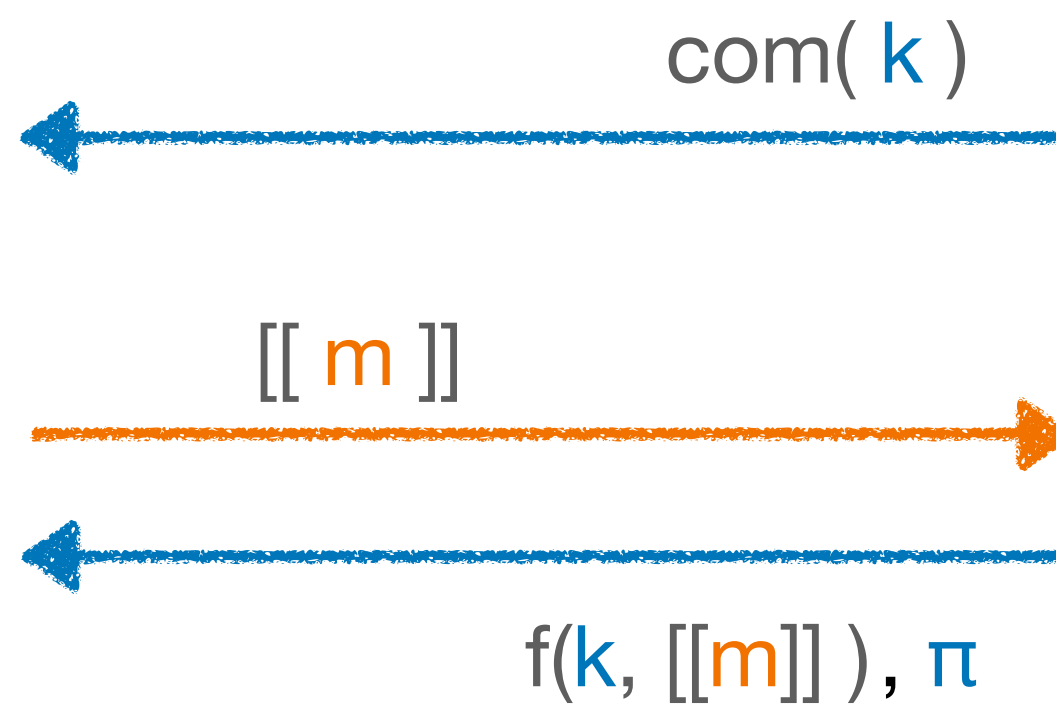


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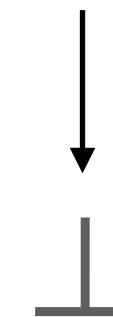


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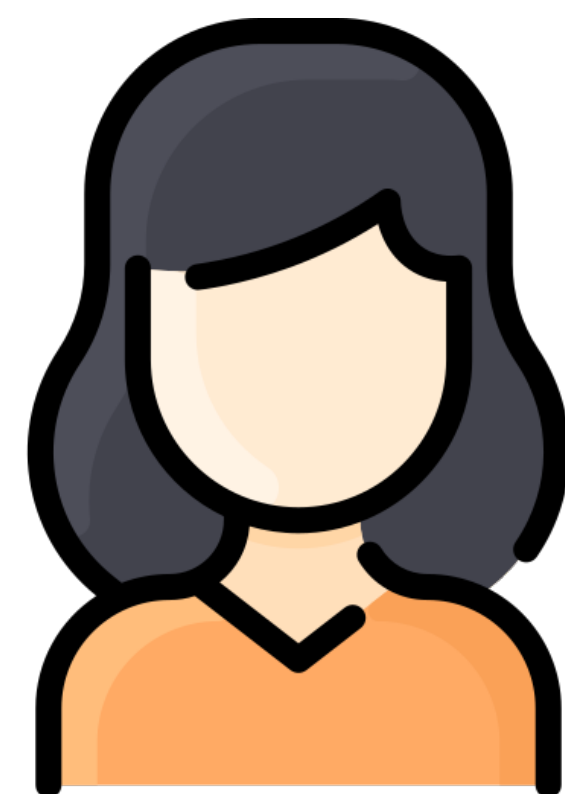
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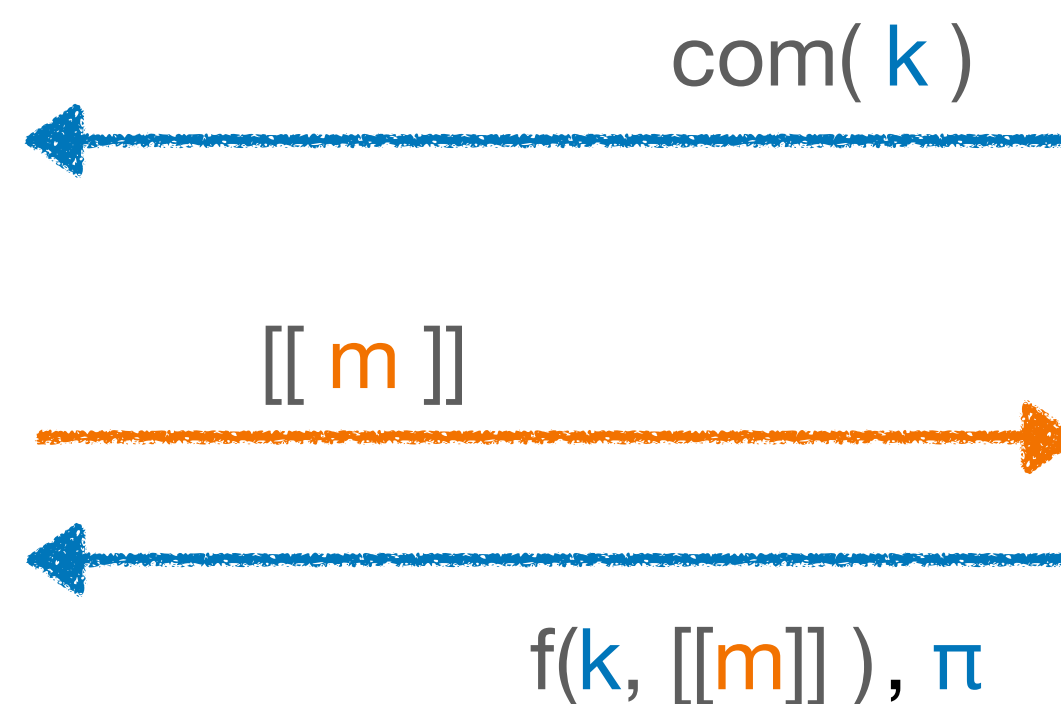


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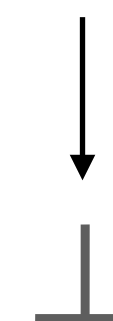


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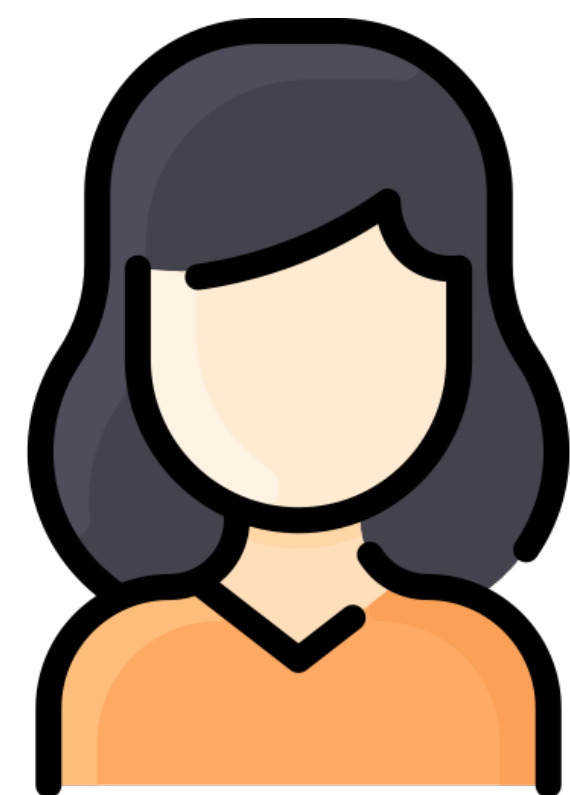


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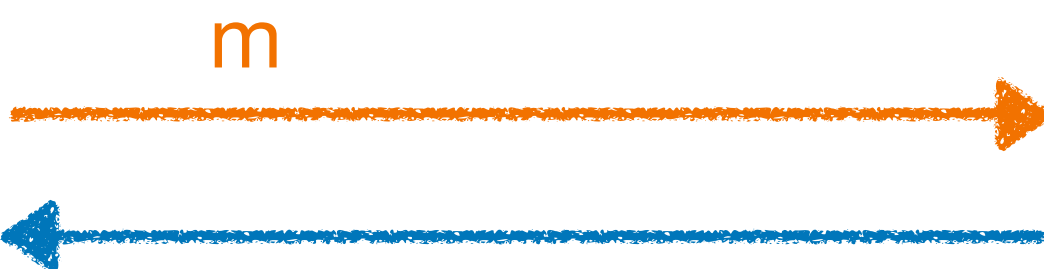


- Password-checking in Microsoft Edge
- OPAQUE
- Privacy pass
- Private-set intersection
- Adaptive OT
- ....

# HashDH OPRF

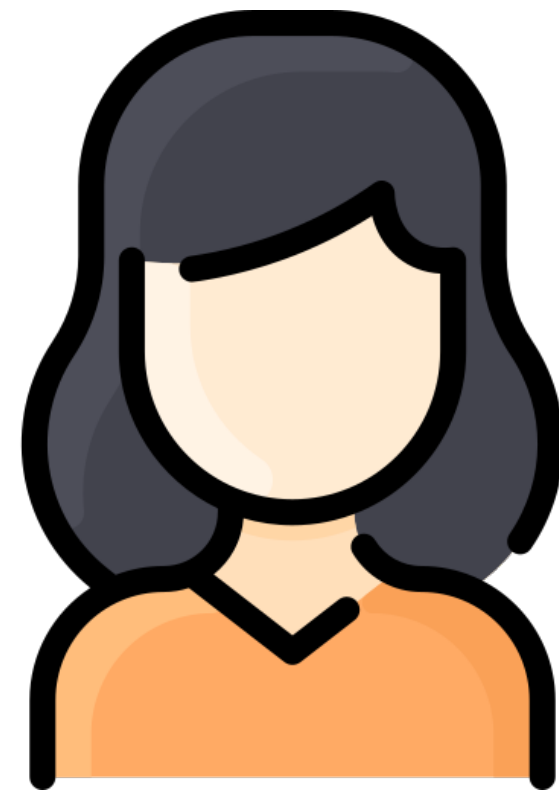


Client

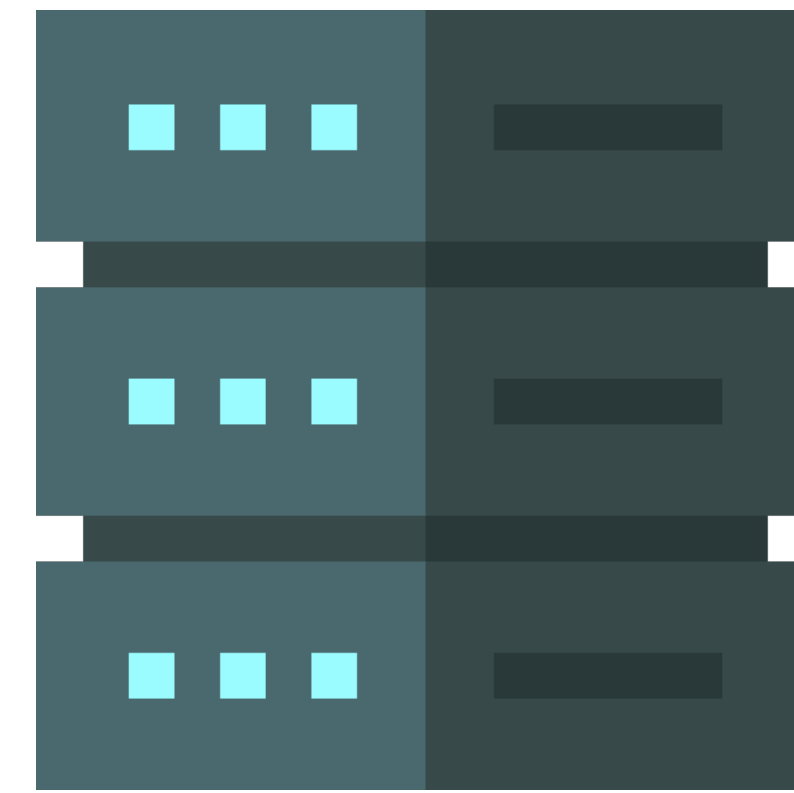
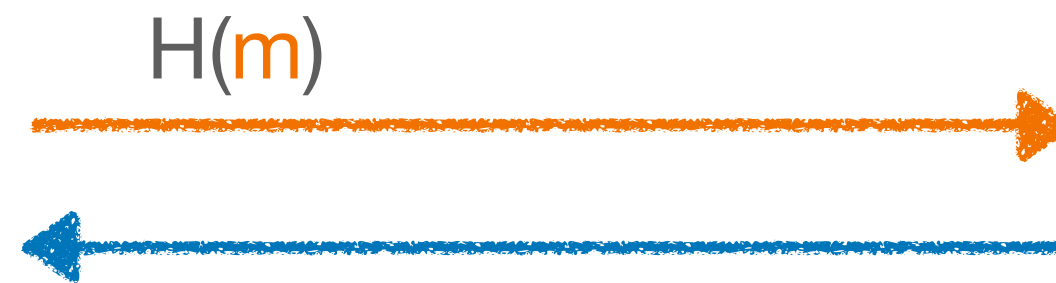


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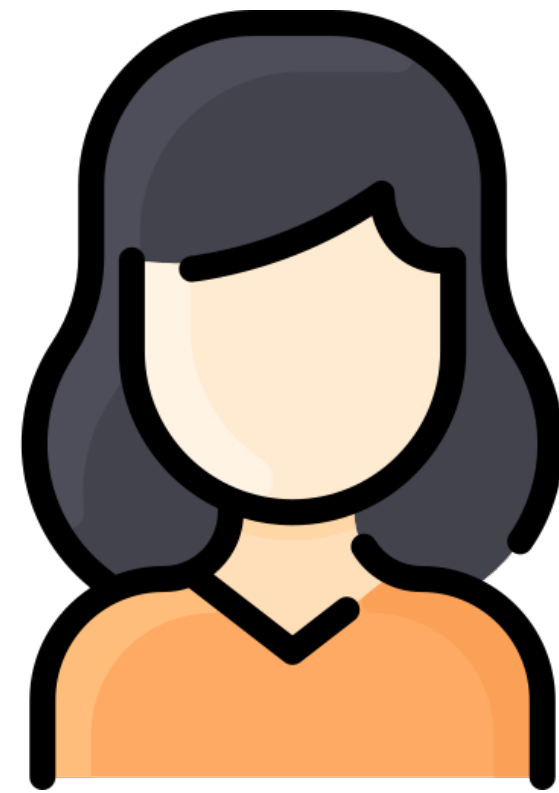


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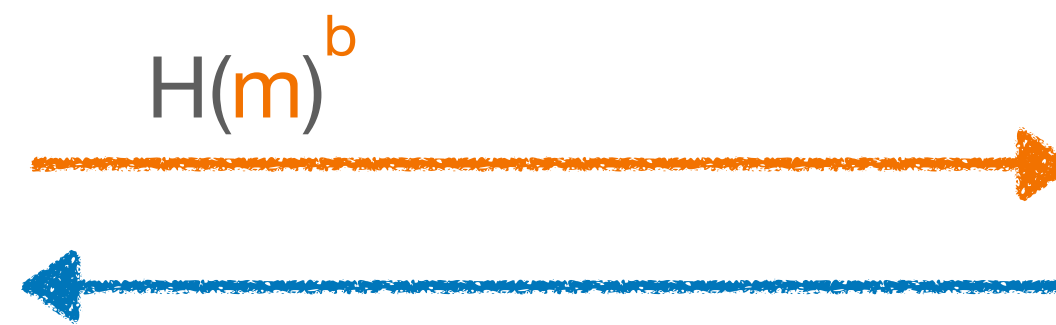


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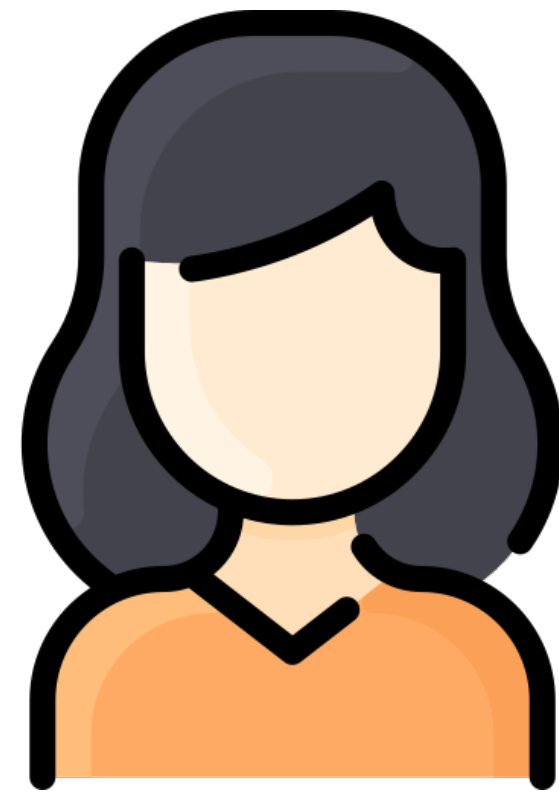


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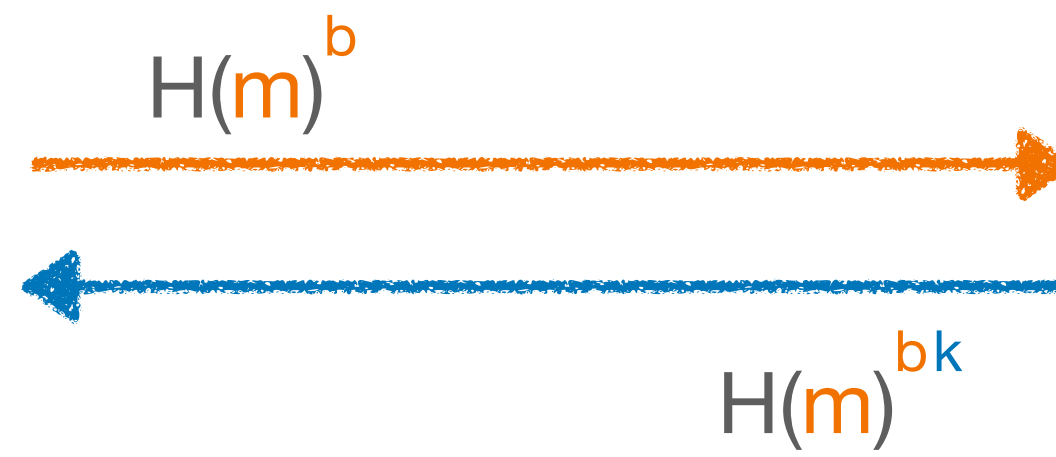


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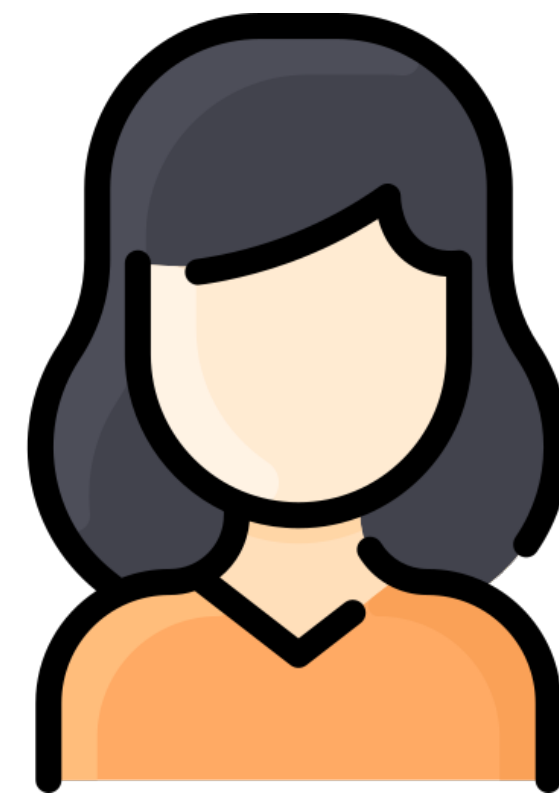


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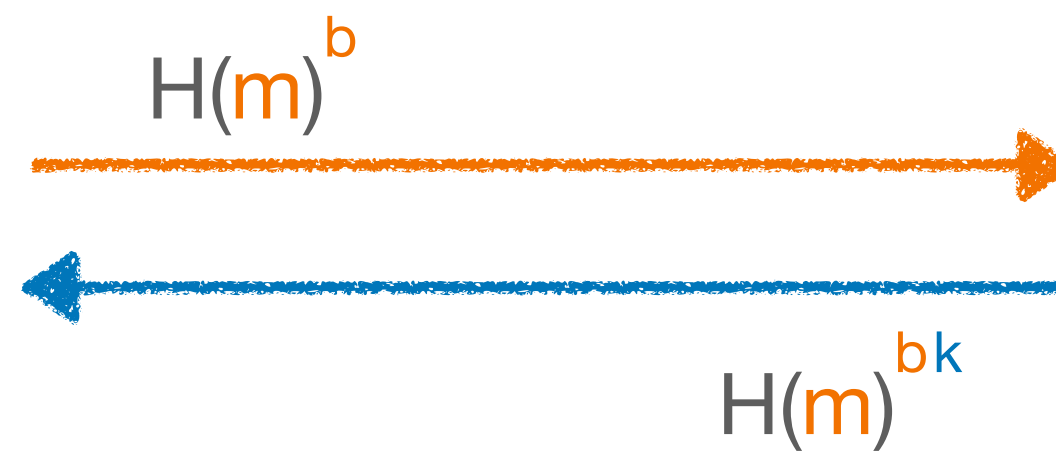
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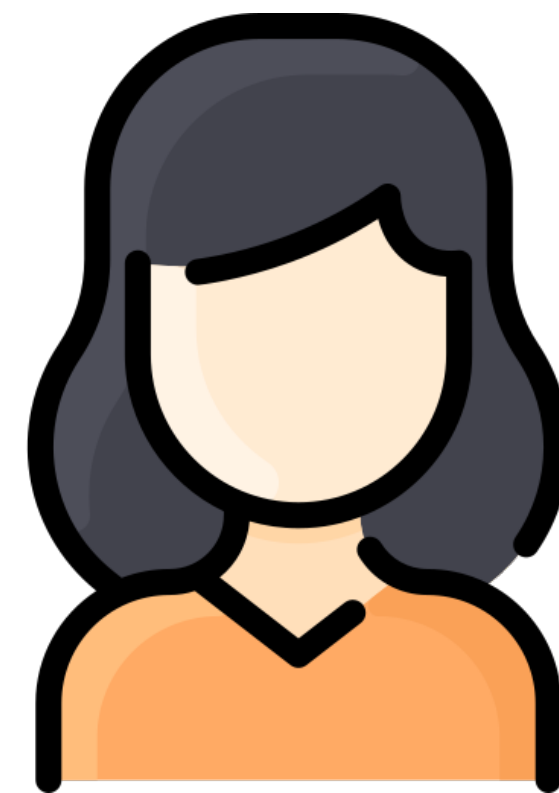


$H(m)^k$



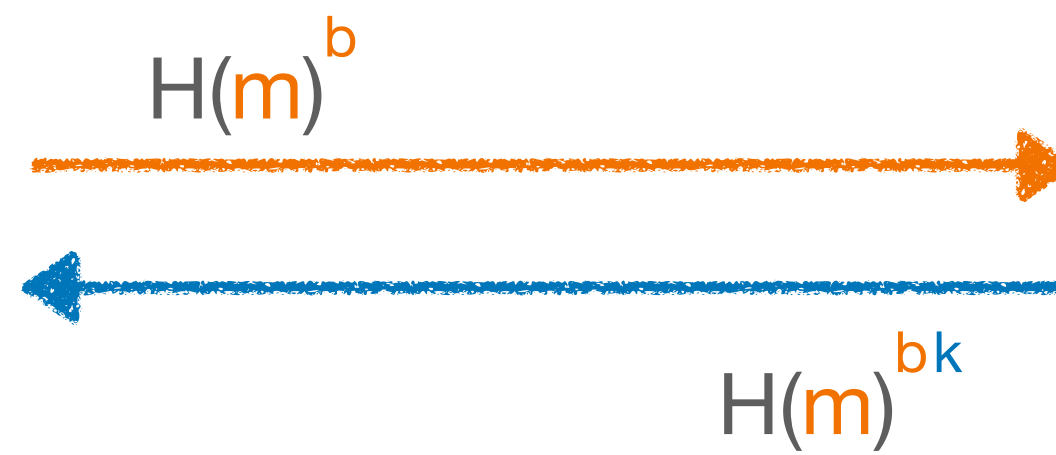
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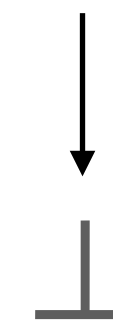


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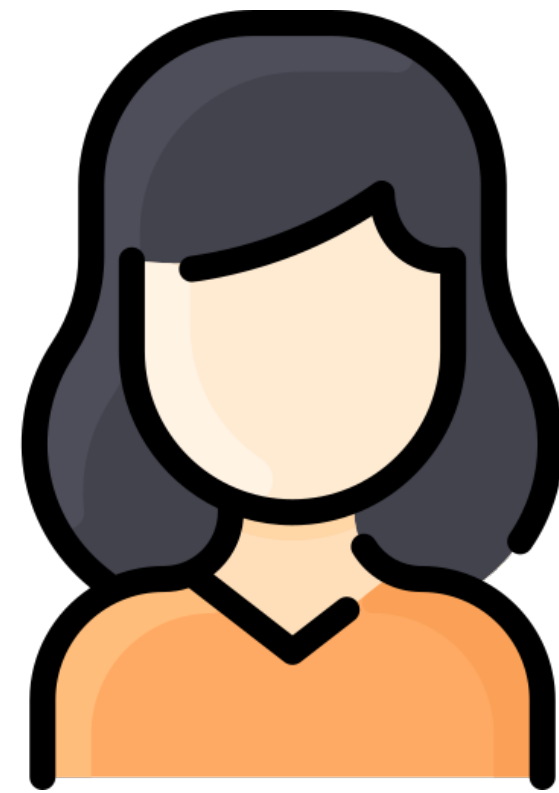
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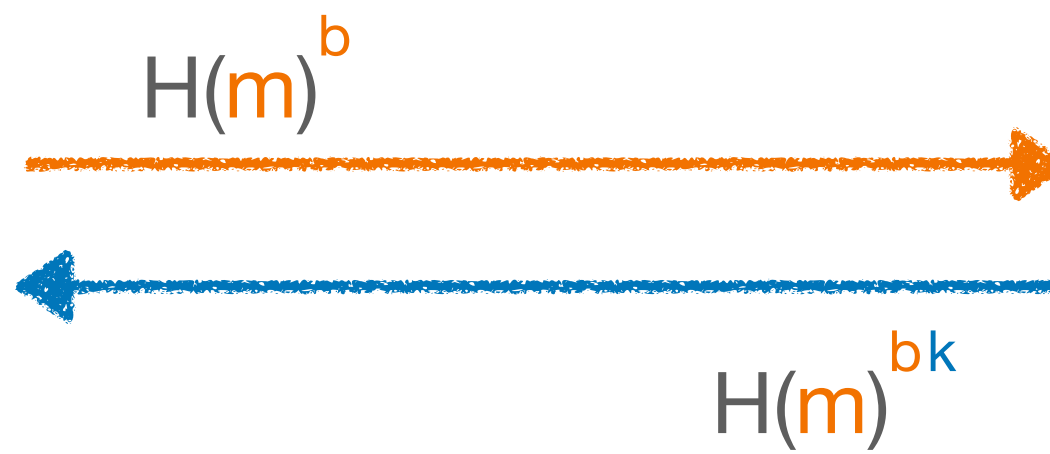


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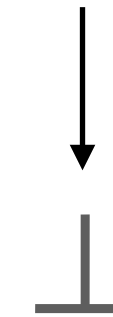


Client

$H(m)^k$



Server



- Server doesn't learn anything ✓
- Output is deterministic ✓
- Client only learns one output ✓

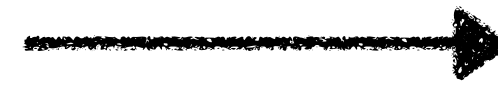
# Post-quantum OPRFs

- Generic MPC techniques



- many rounds (can't be optimal)

- VOPRF based on lattices [ADDs19]



- round optimal
- feasibility result ( $> 2^{40}$  bits of comms)

- VOPRF based on SIDH [BKW20]



- six rounds
- broken by attack on PR and on SIDH

- OPRF based on CSIDH [BKW20]



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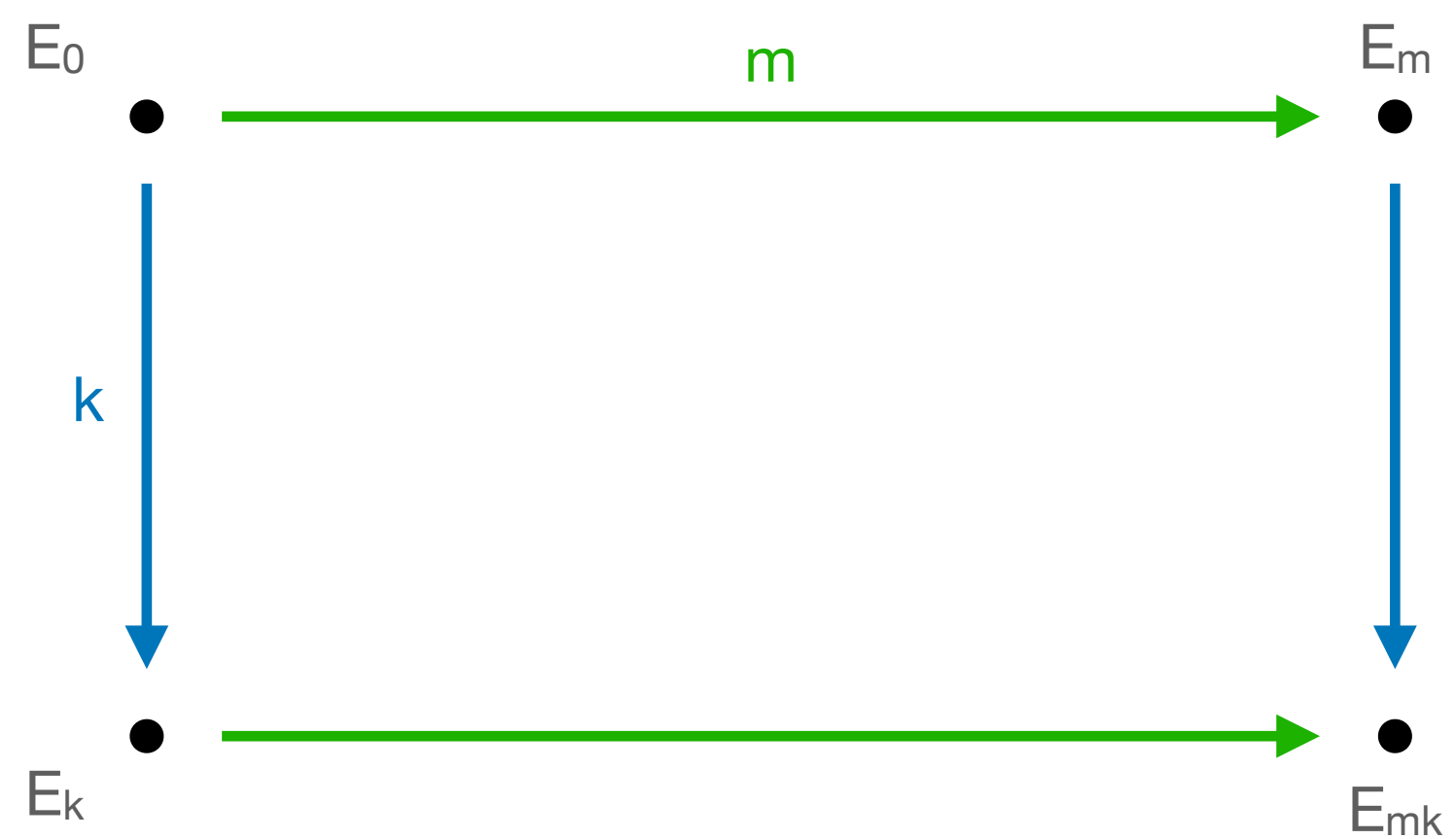
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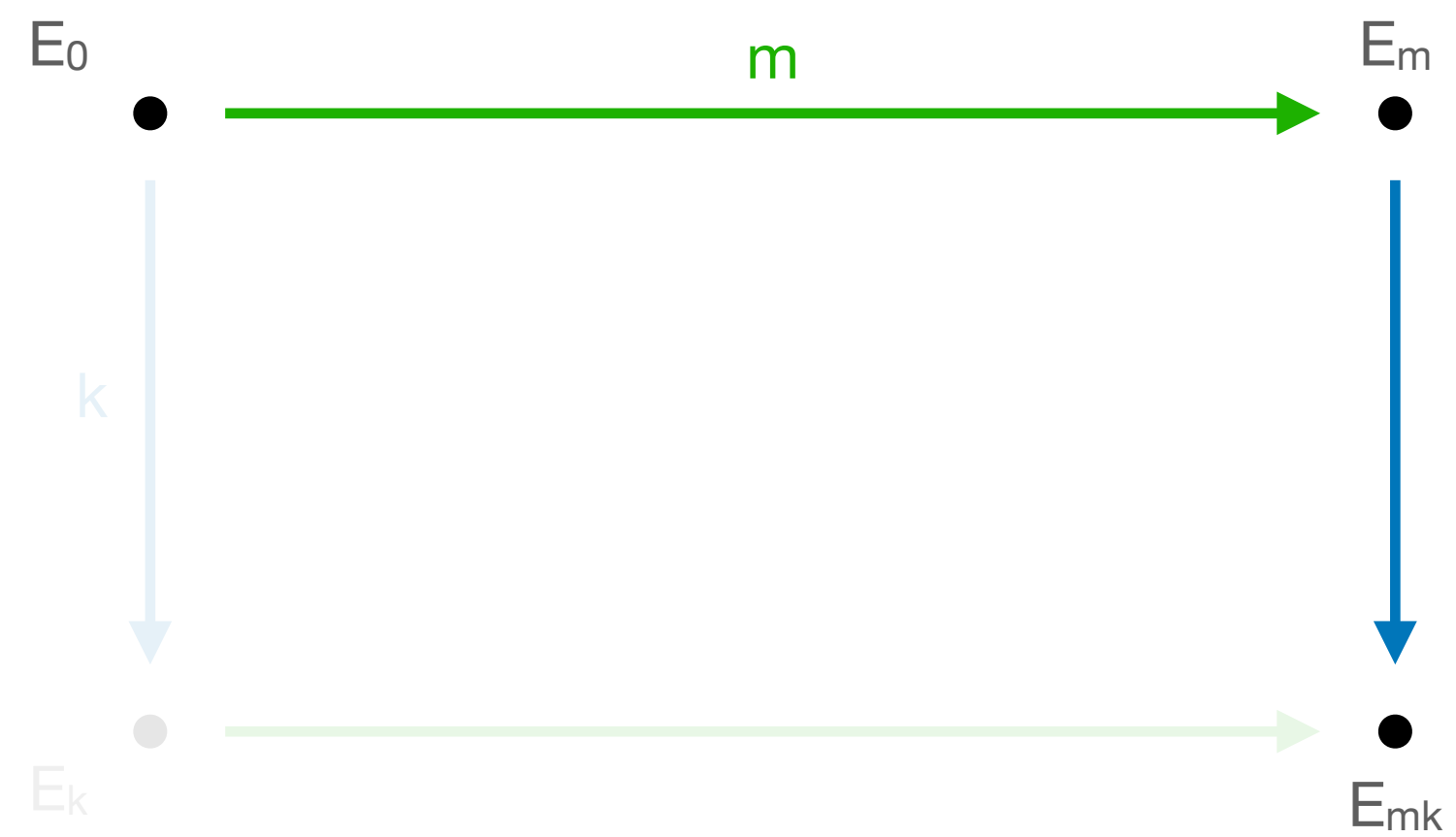


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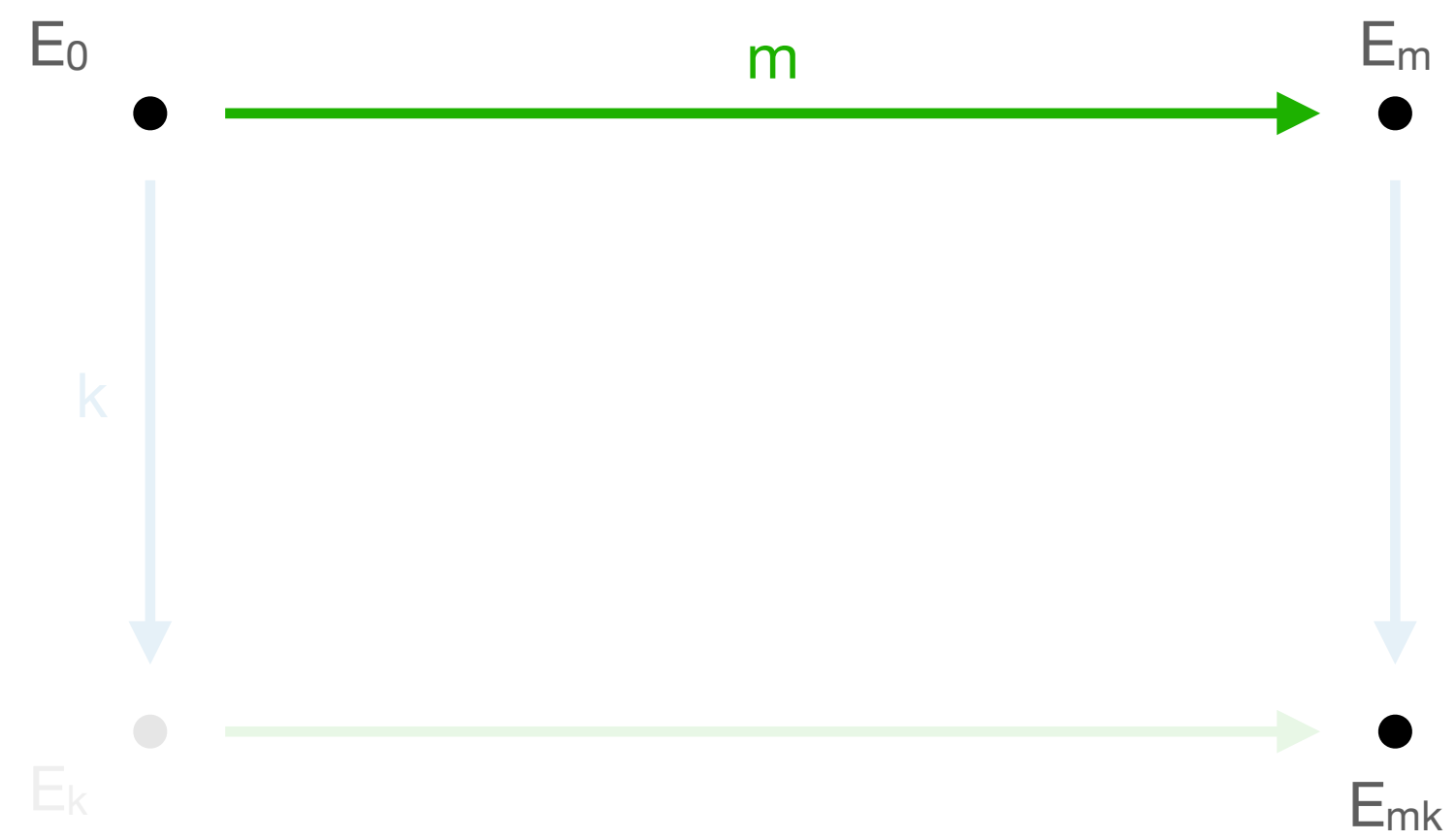
# The original OPRF [BKW20]



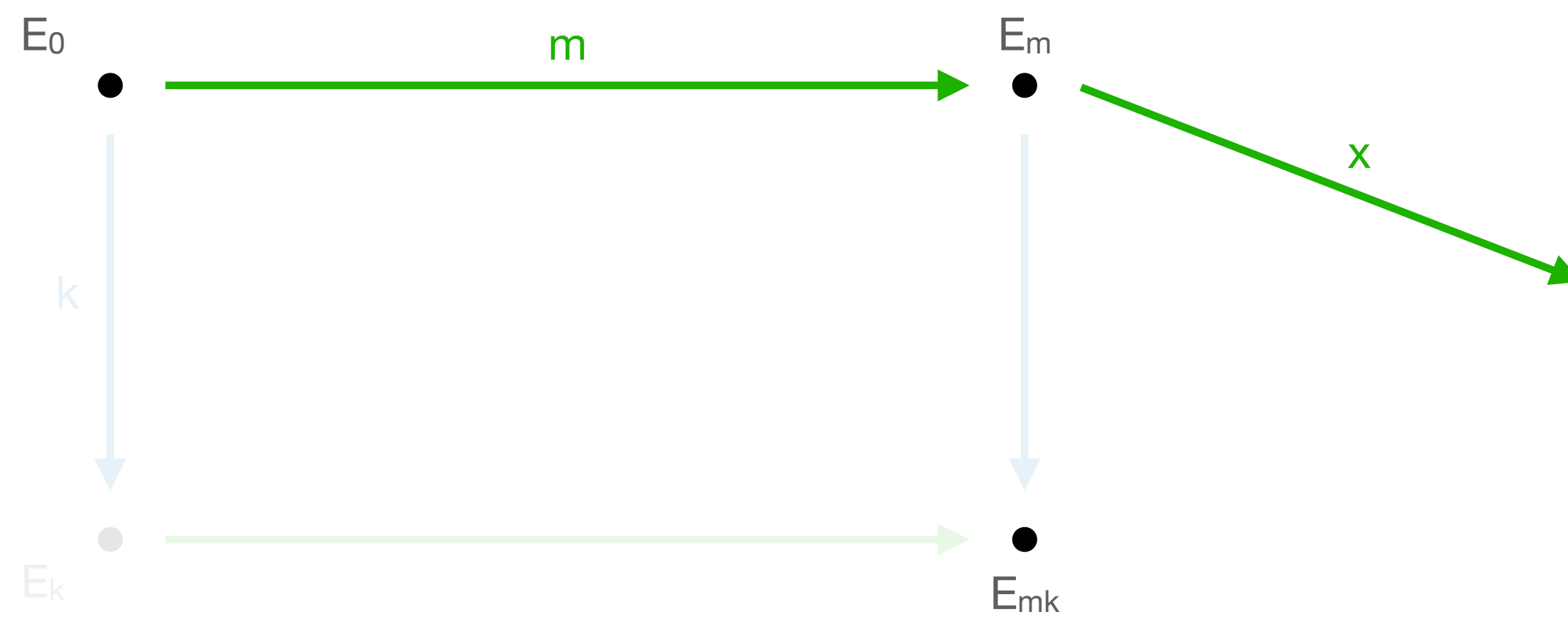
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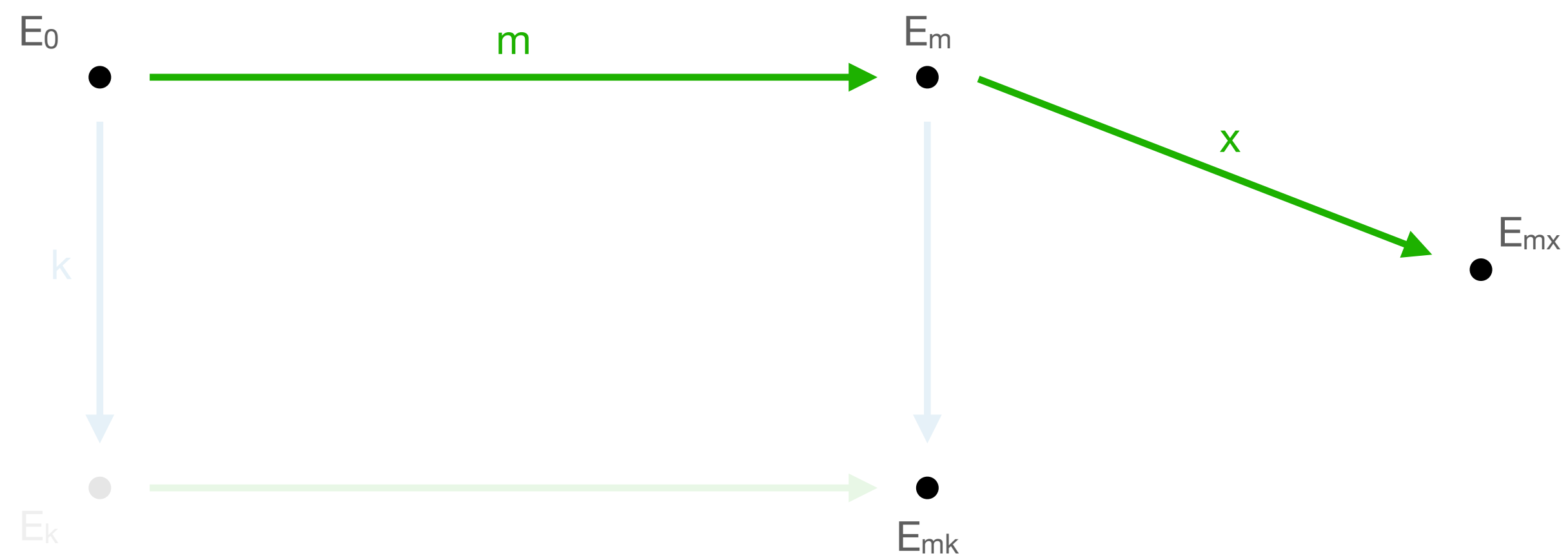
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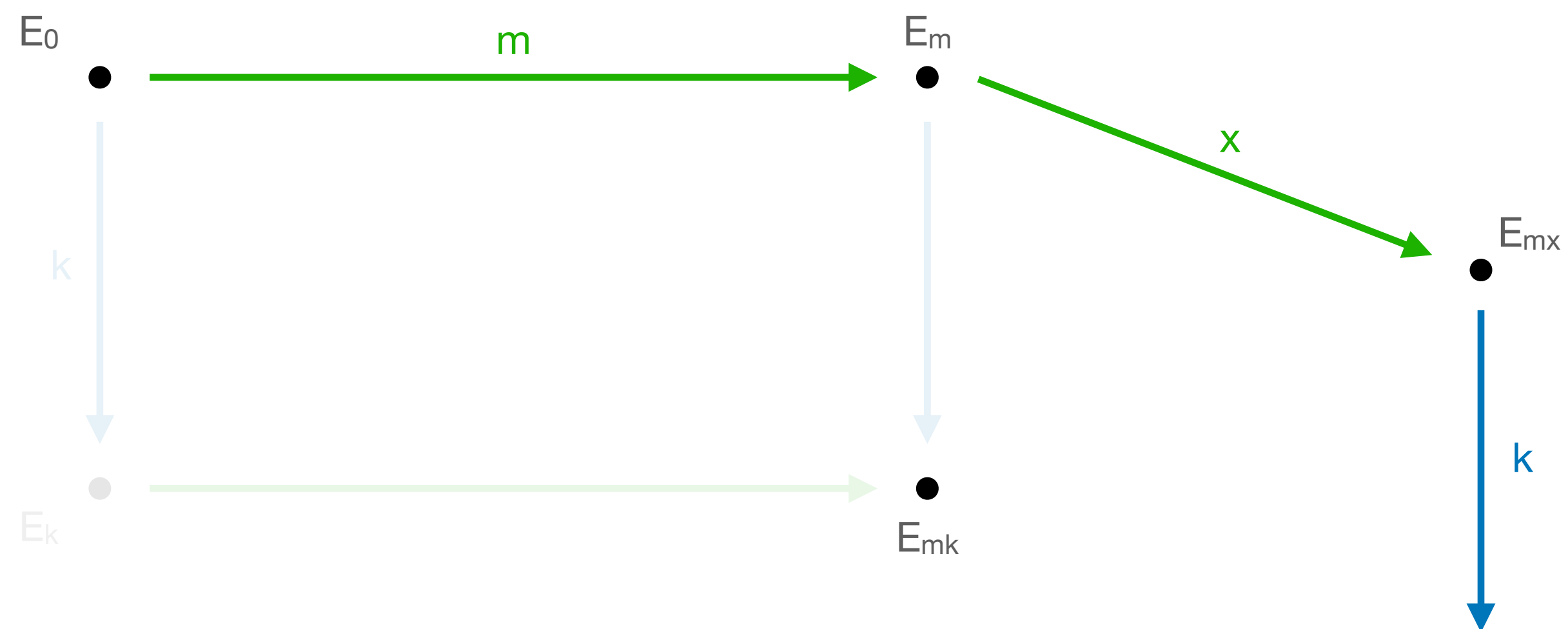
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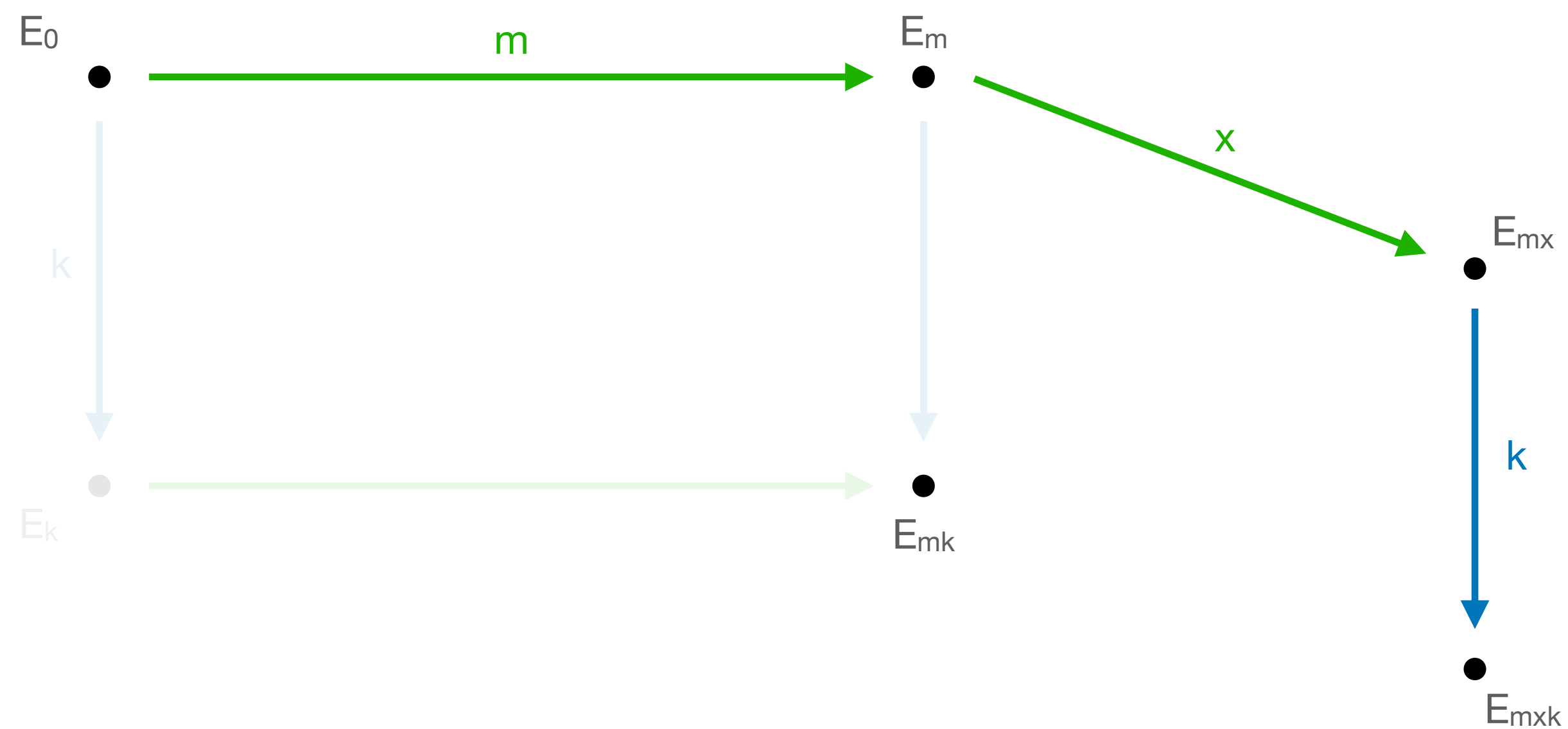
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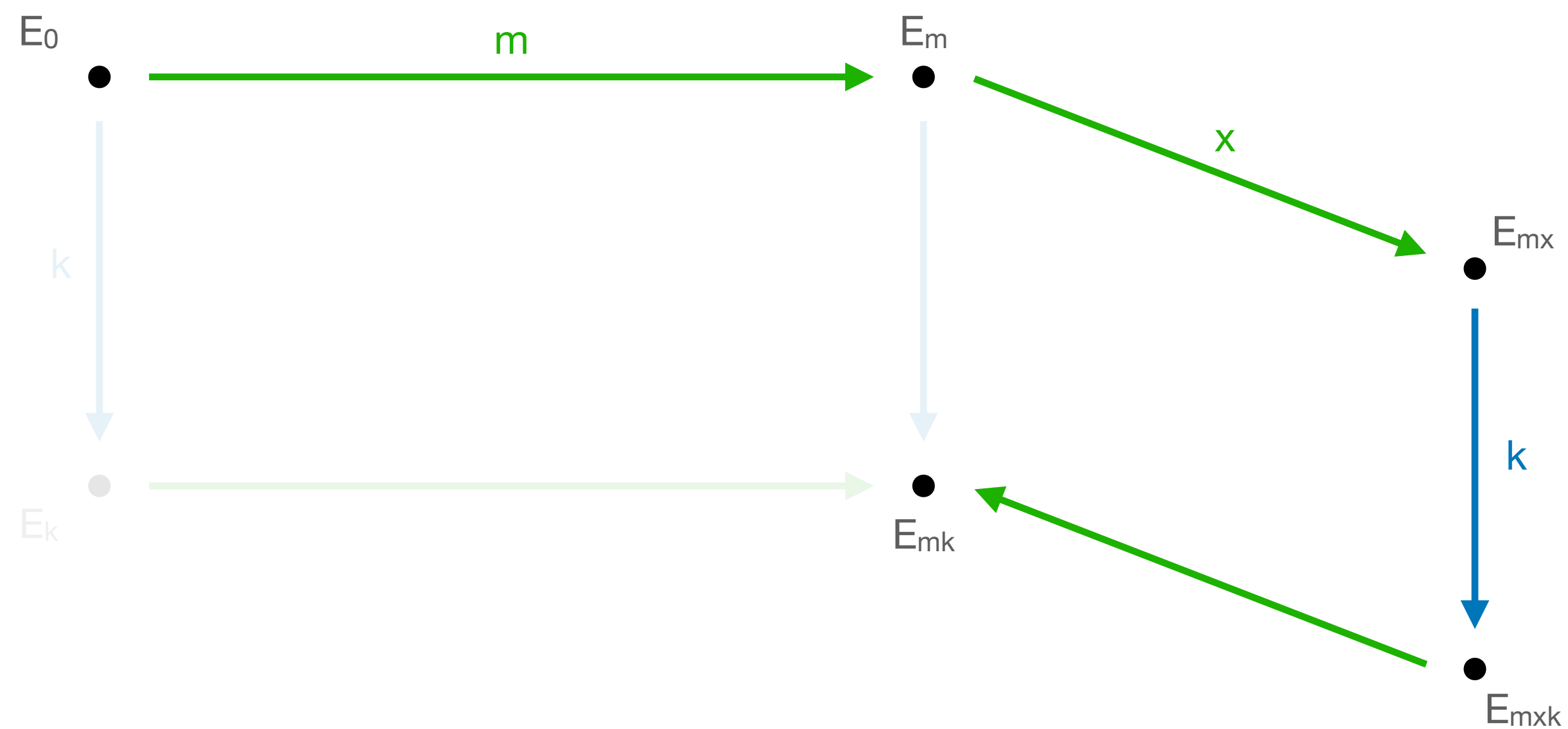
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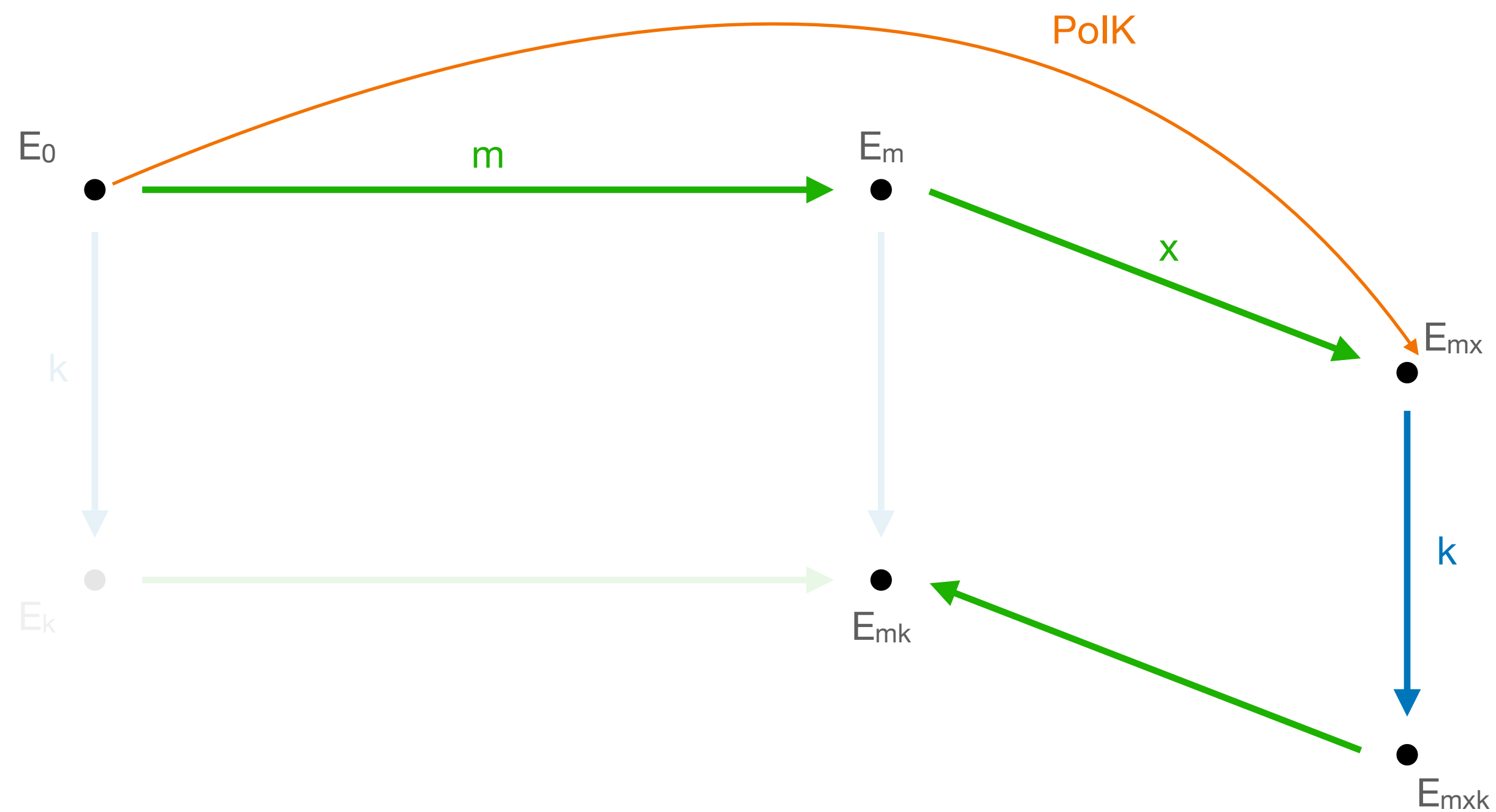
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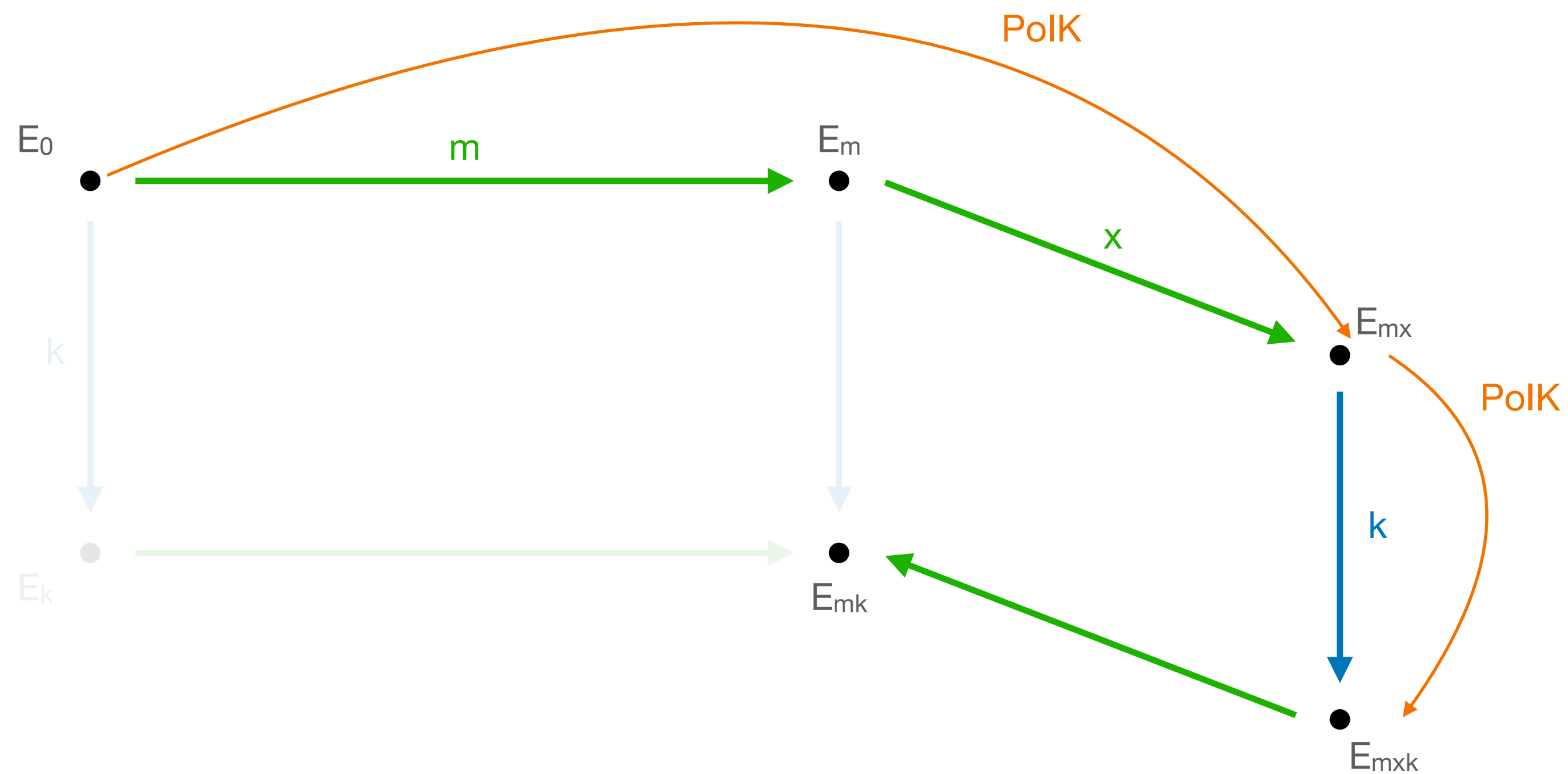
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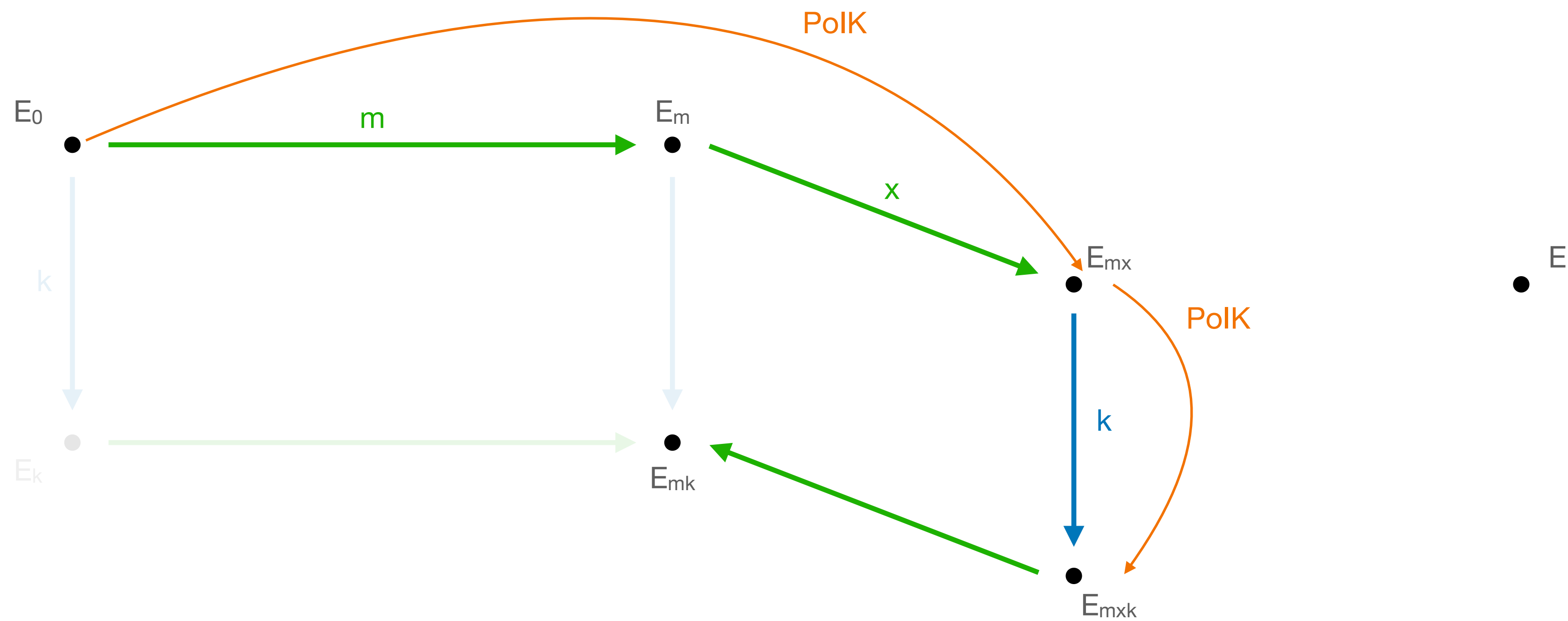
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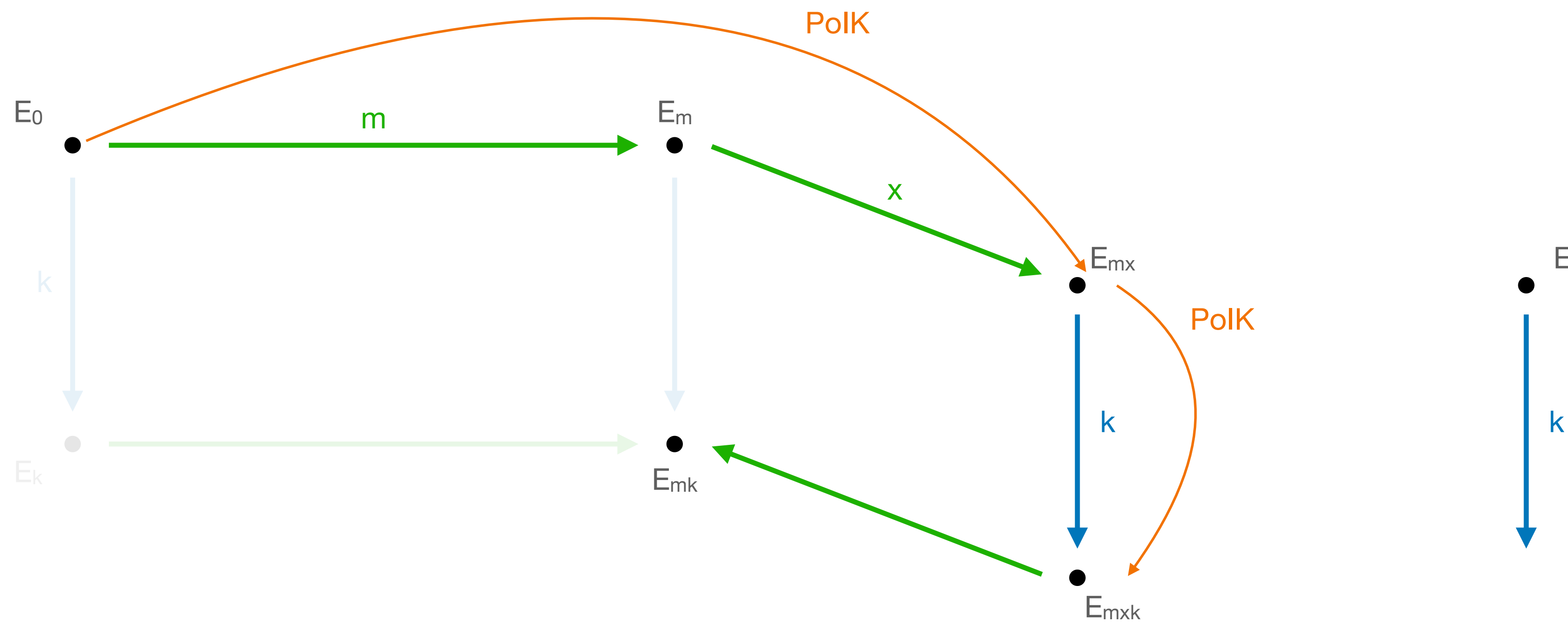
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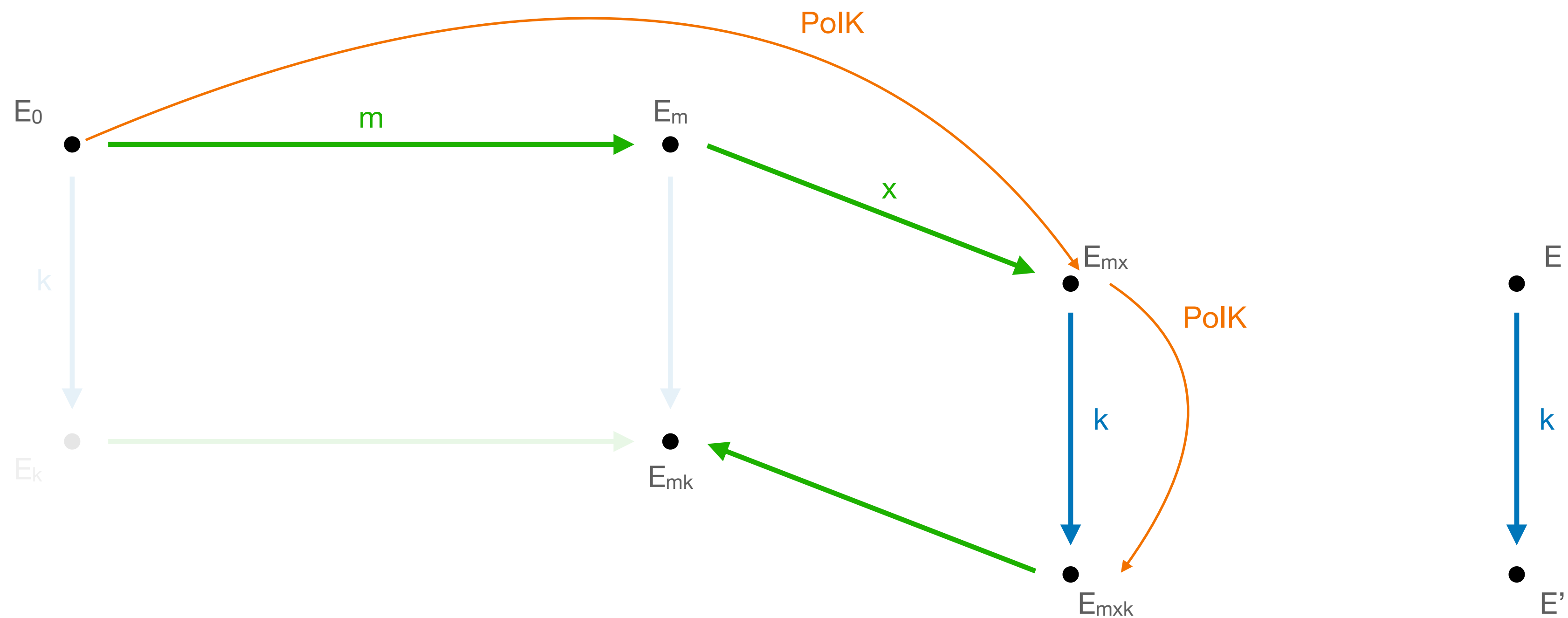
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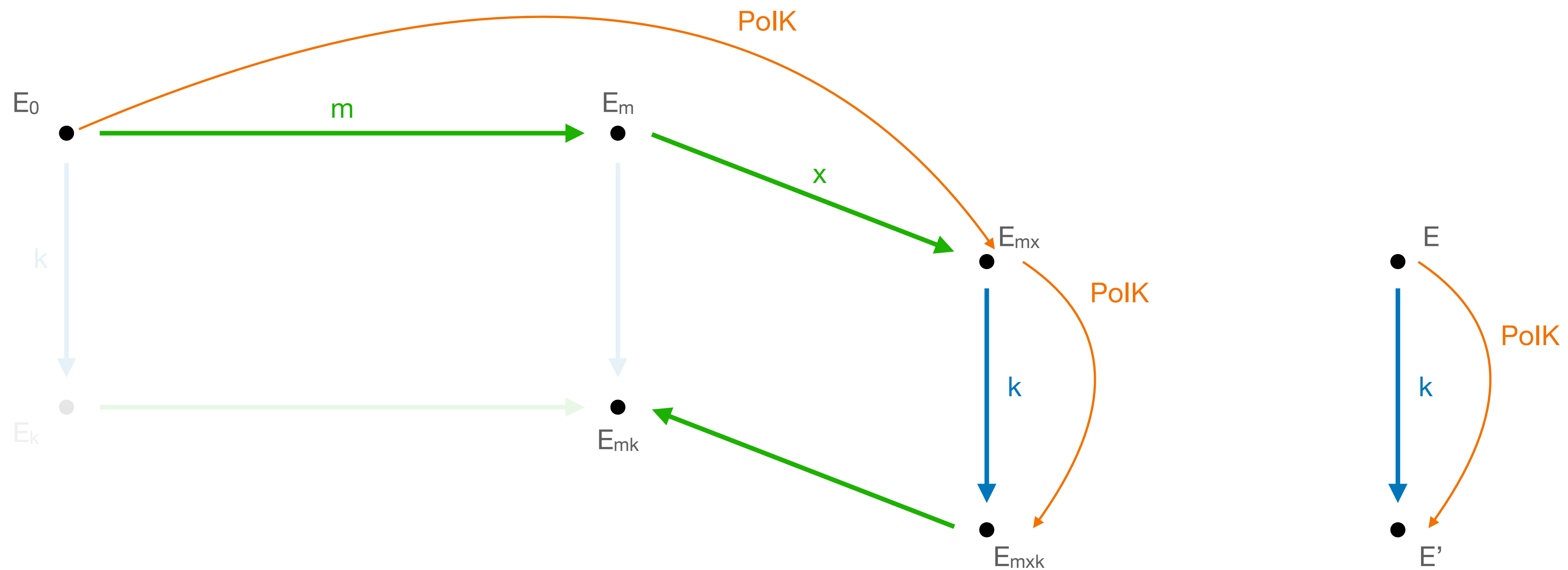
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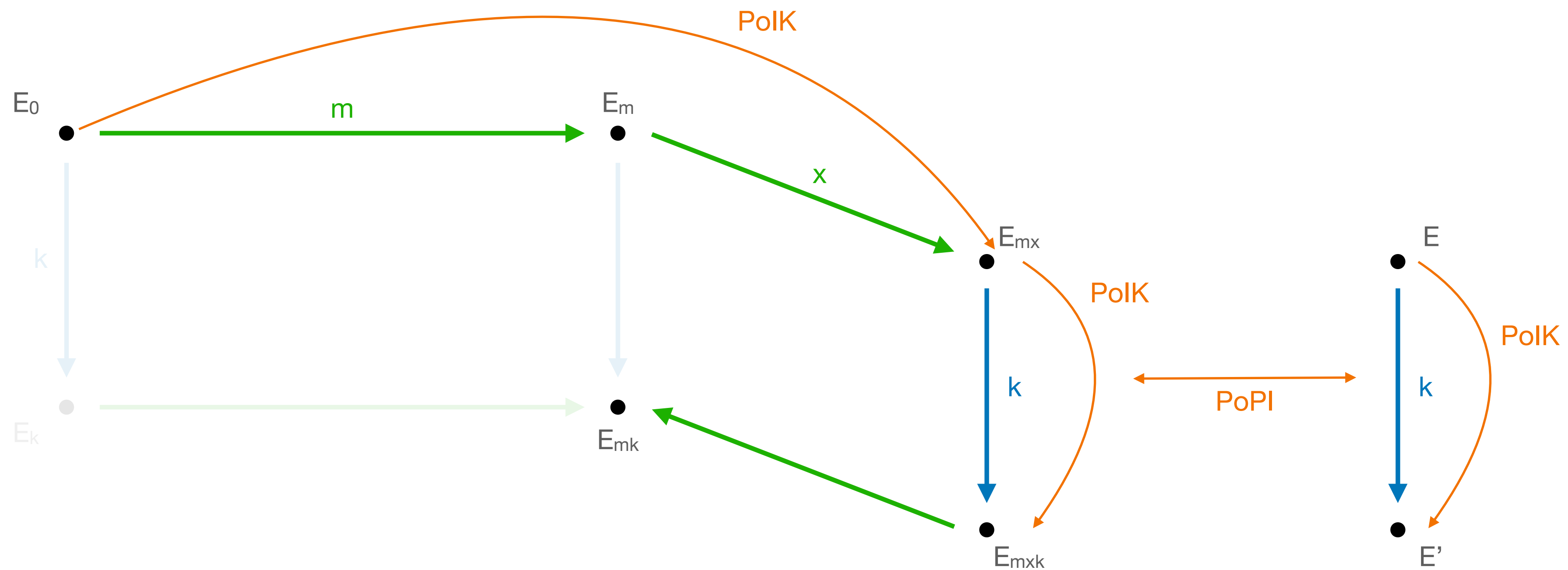
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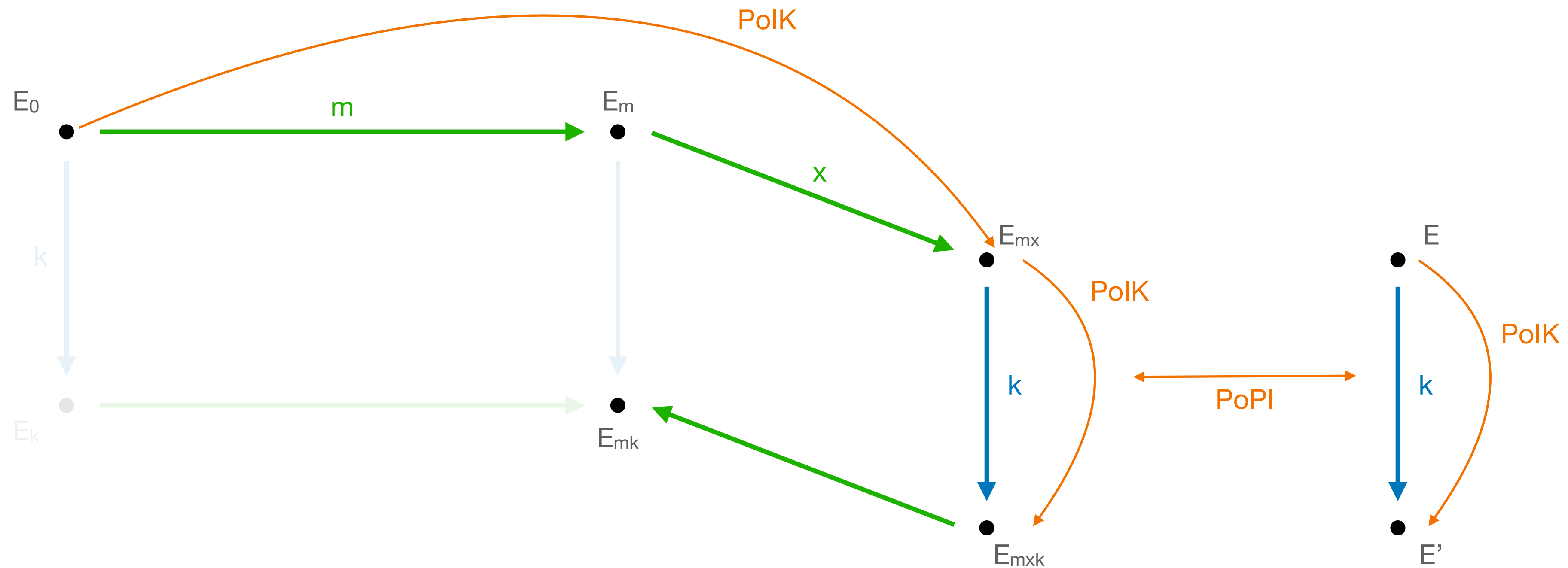
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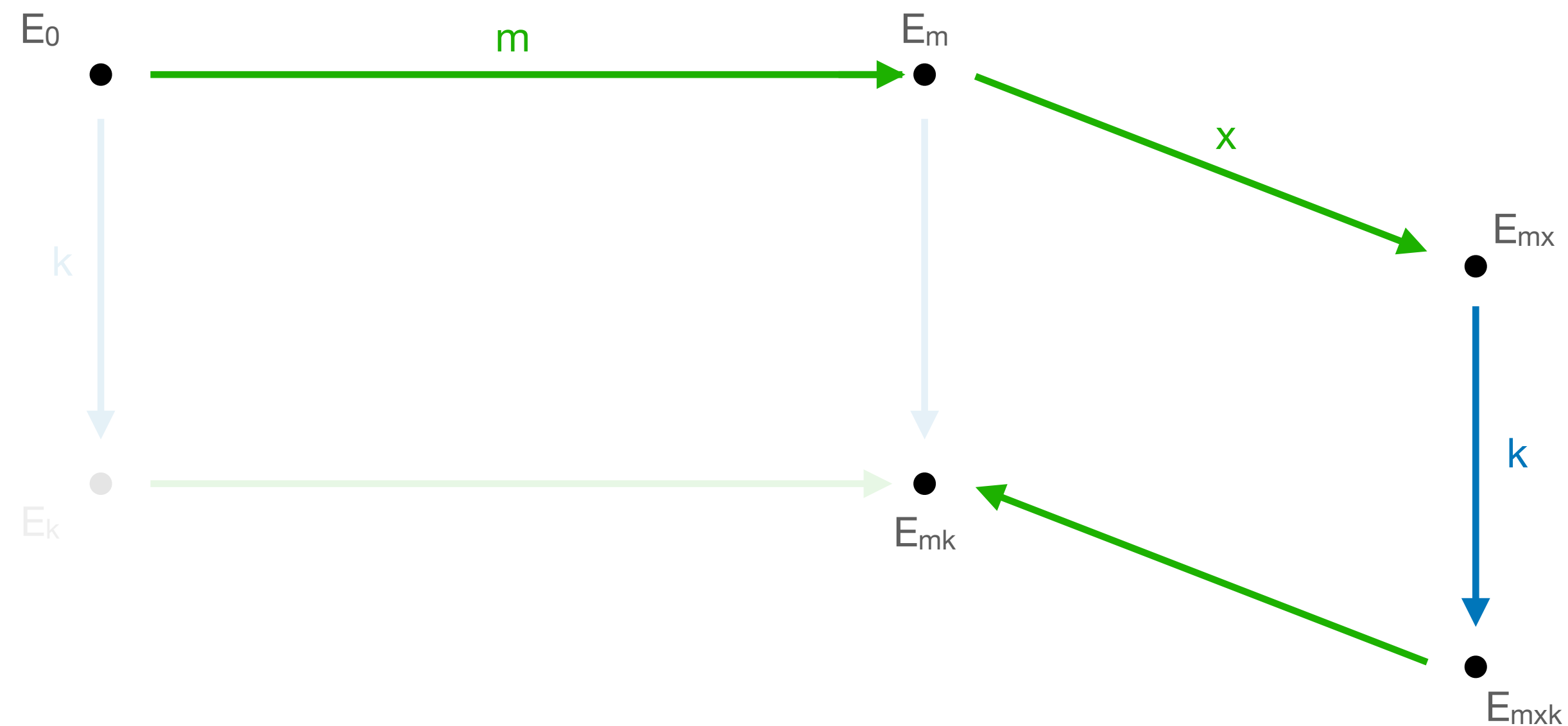
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$$F(k, m) = H(m, j_{mk}, E')$$

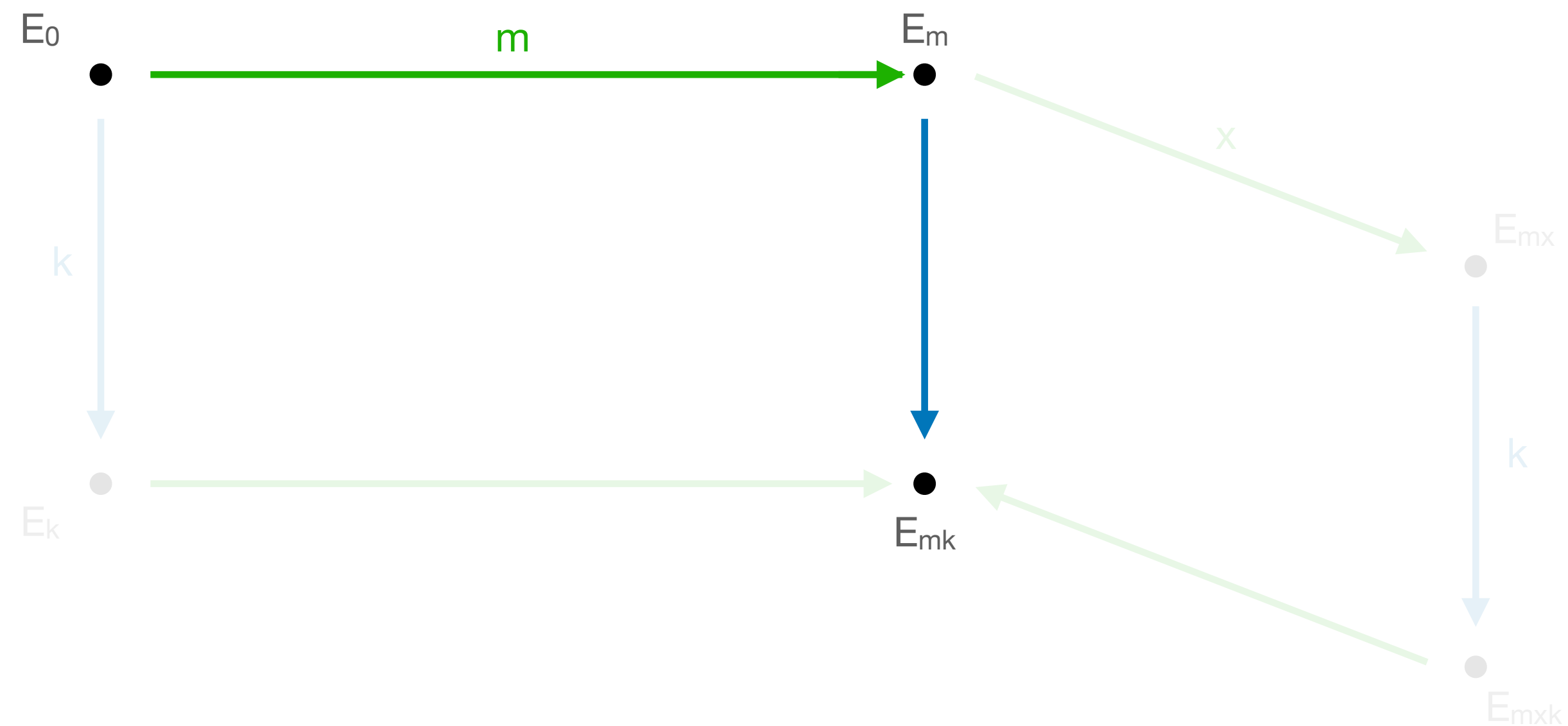
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**Pseudorandomness:** after  $n$  interactions, an attacker cannot generate  $n+1$  PRF outputs



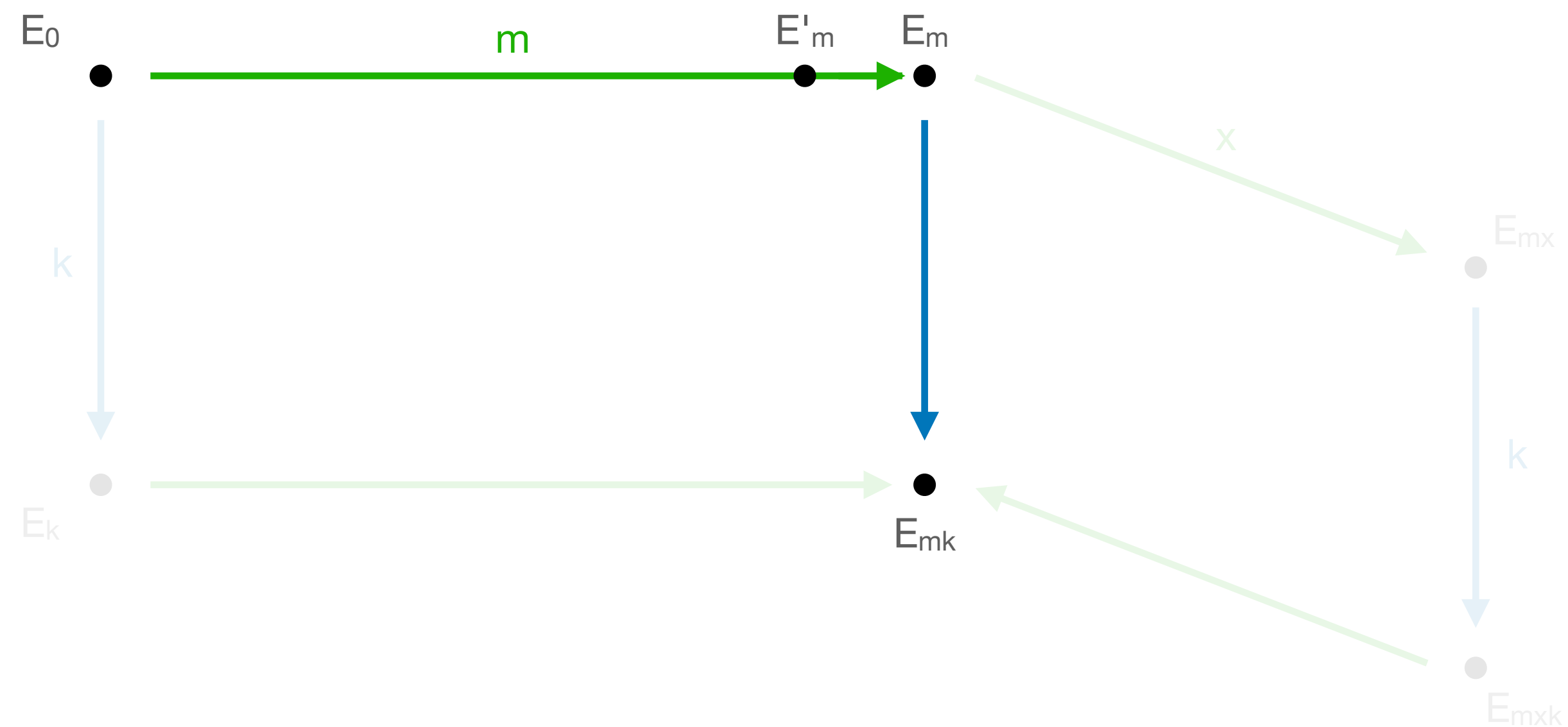
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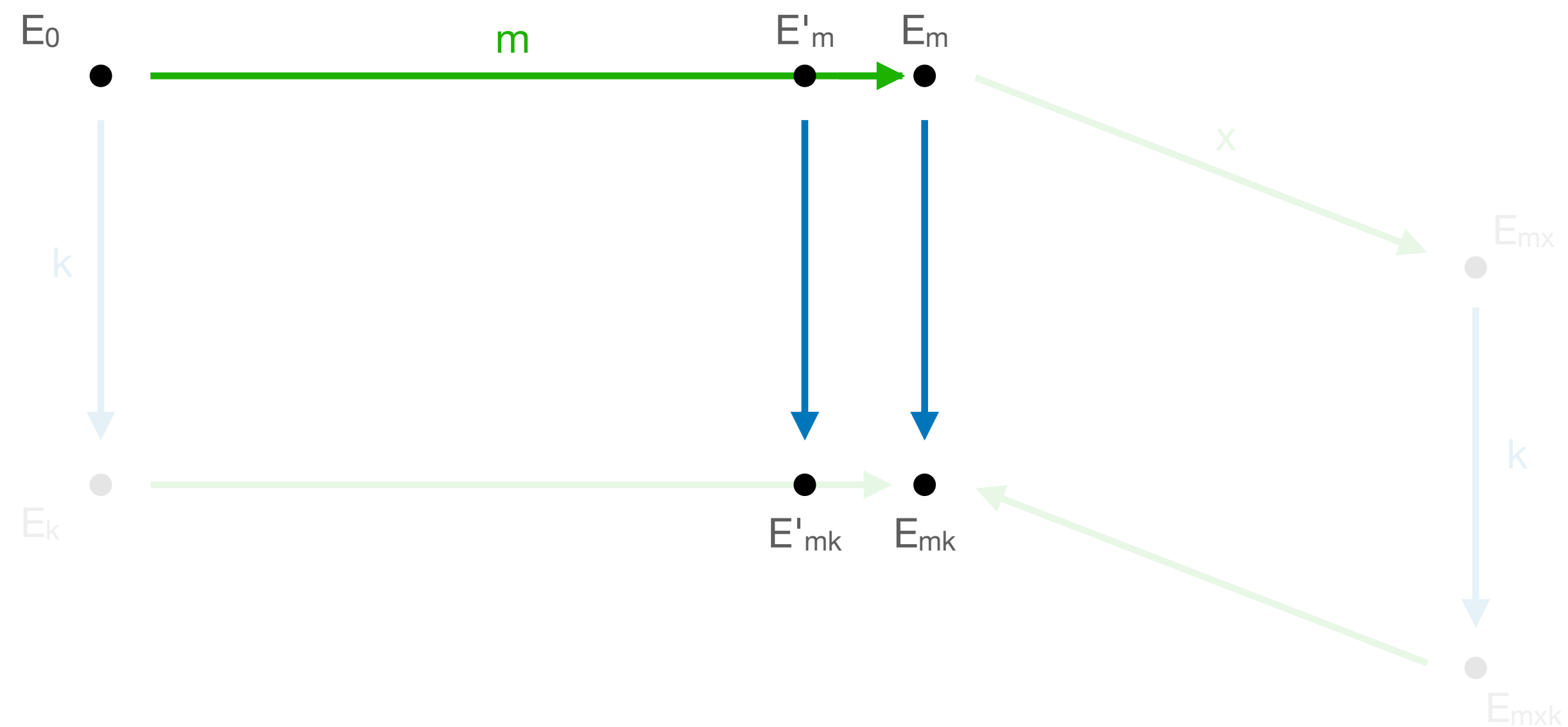
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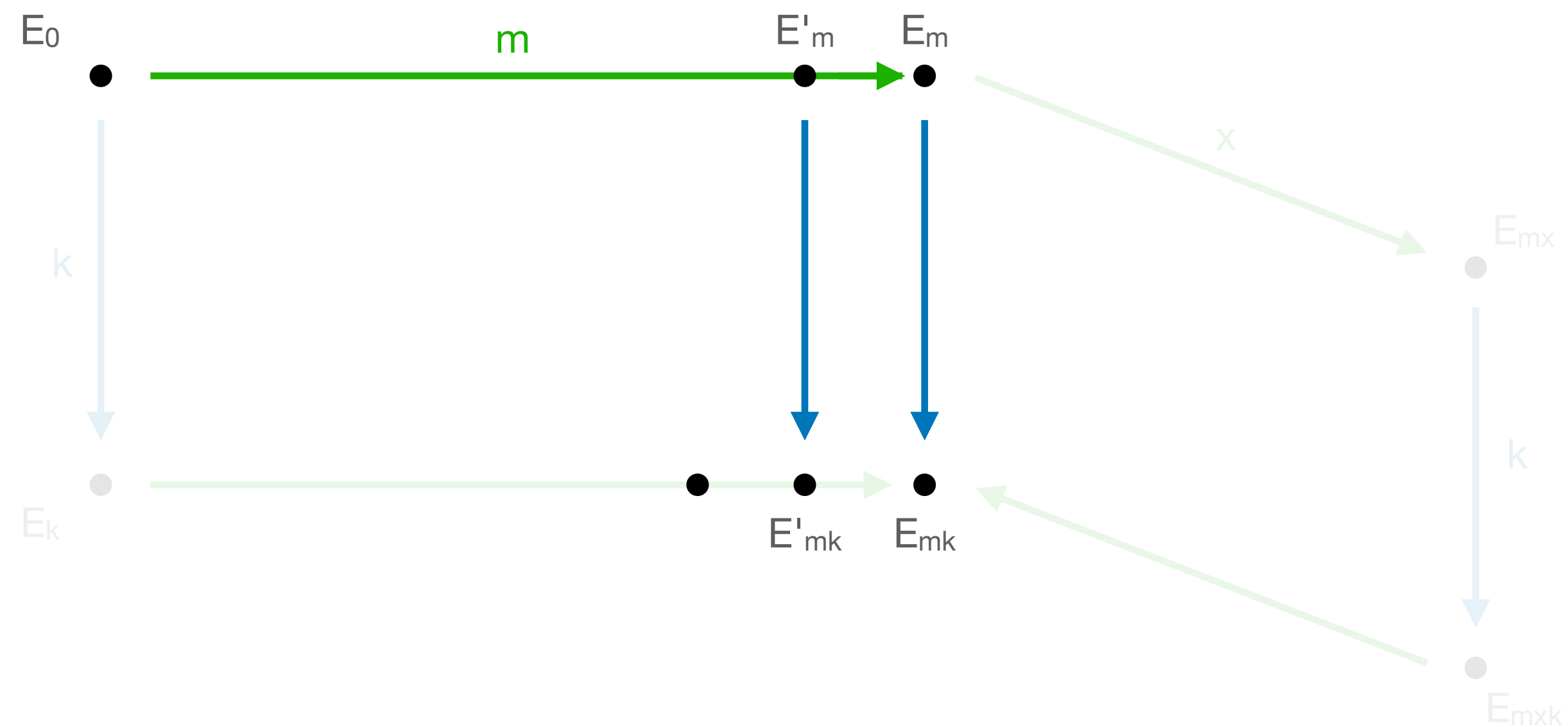
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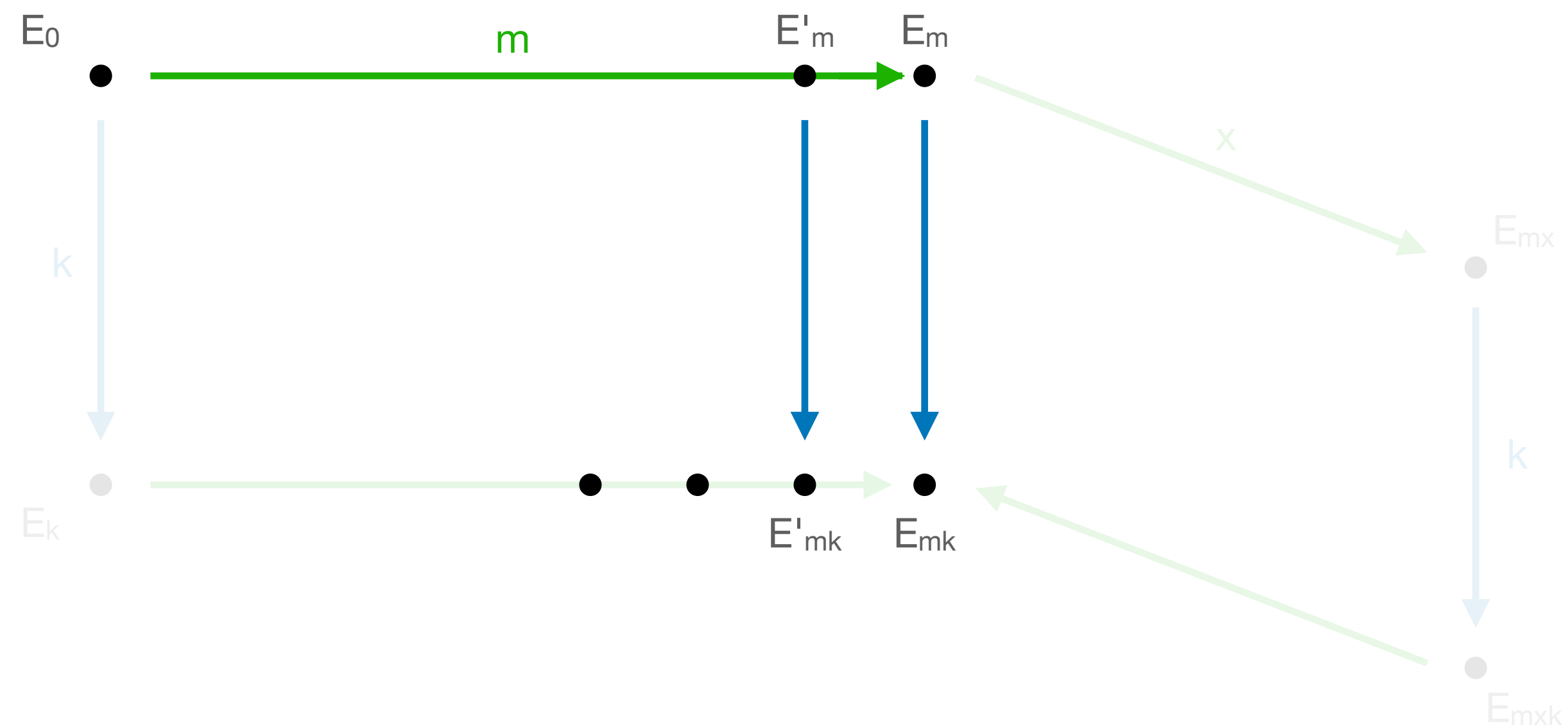
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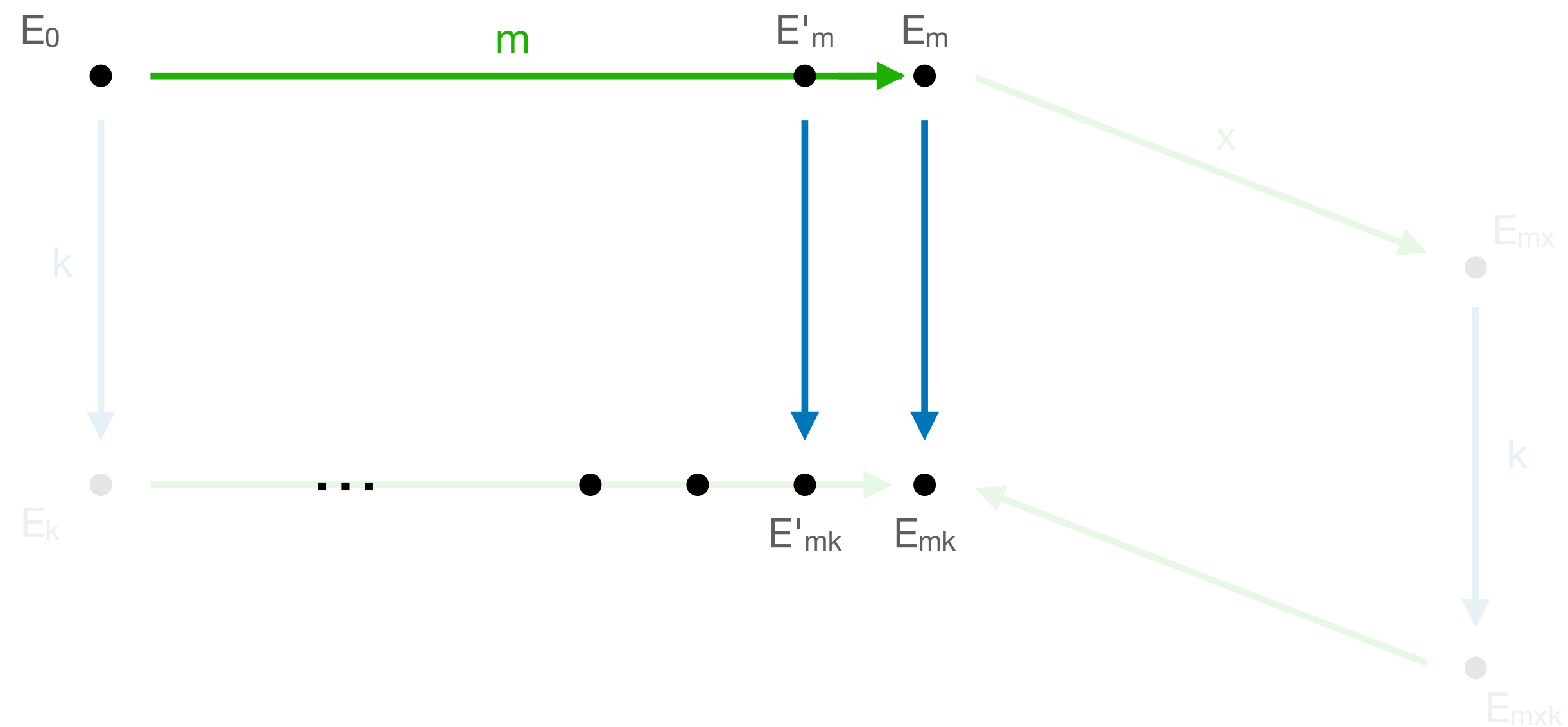
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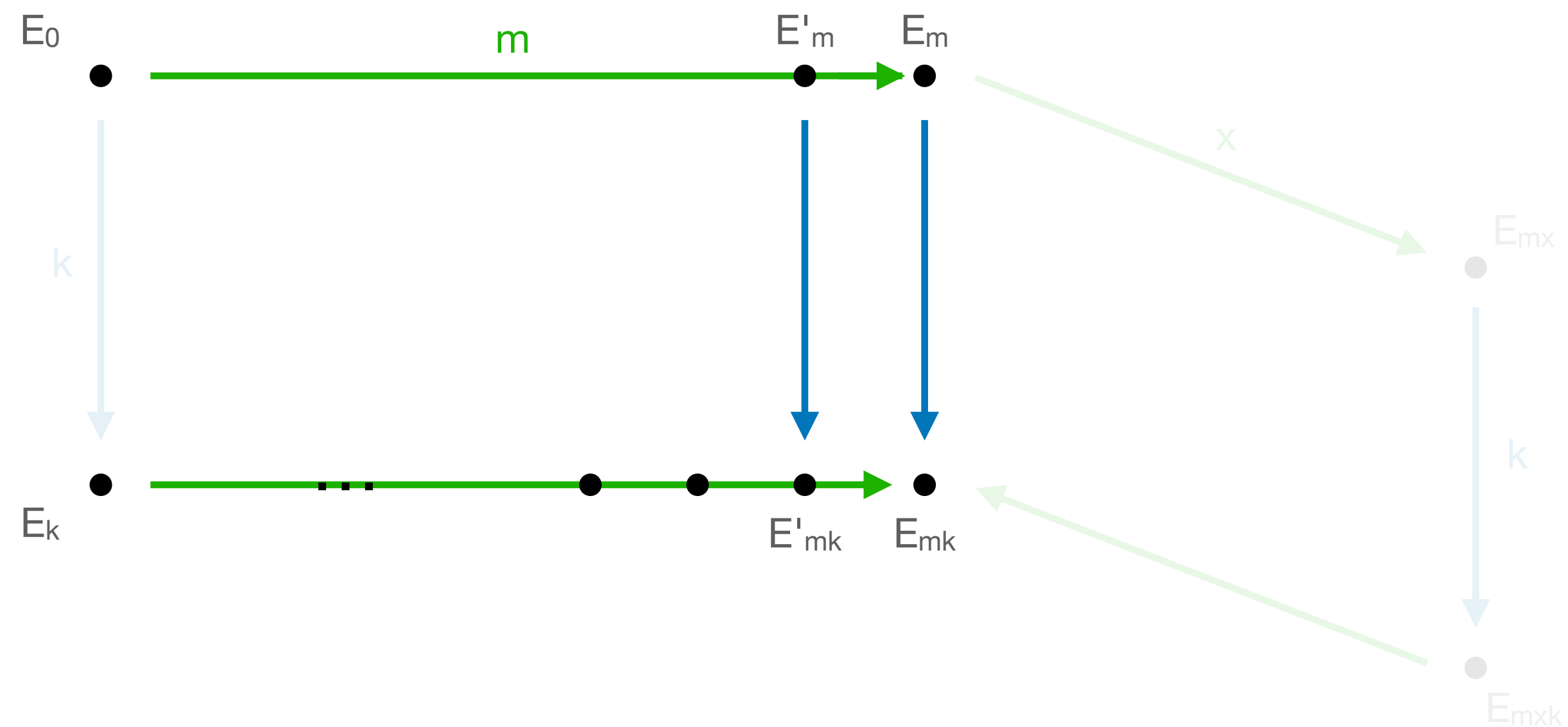
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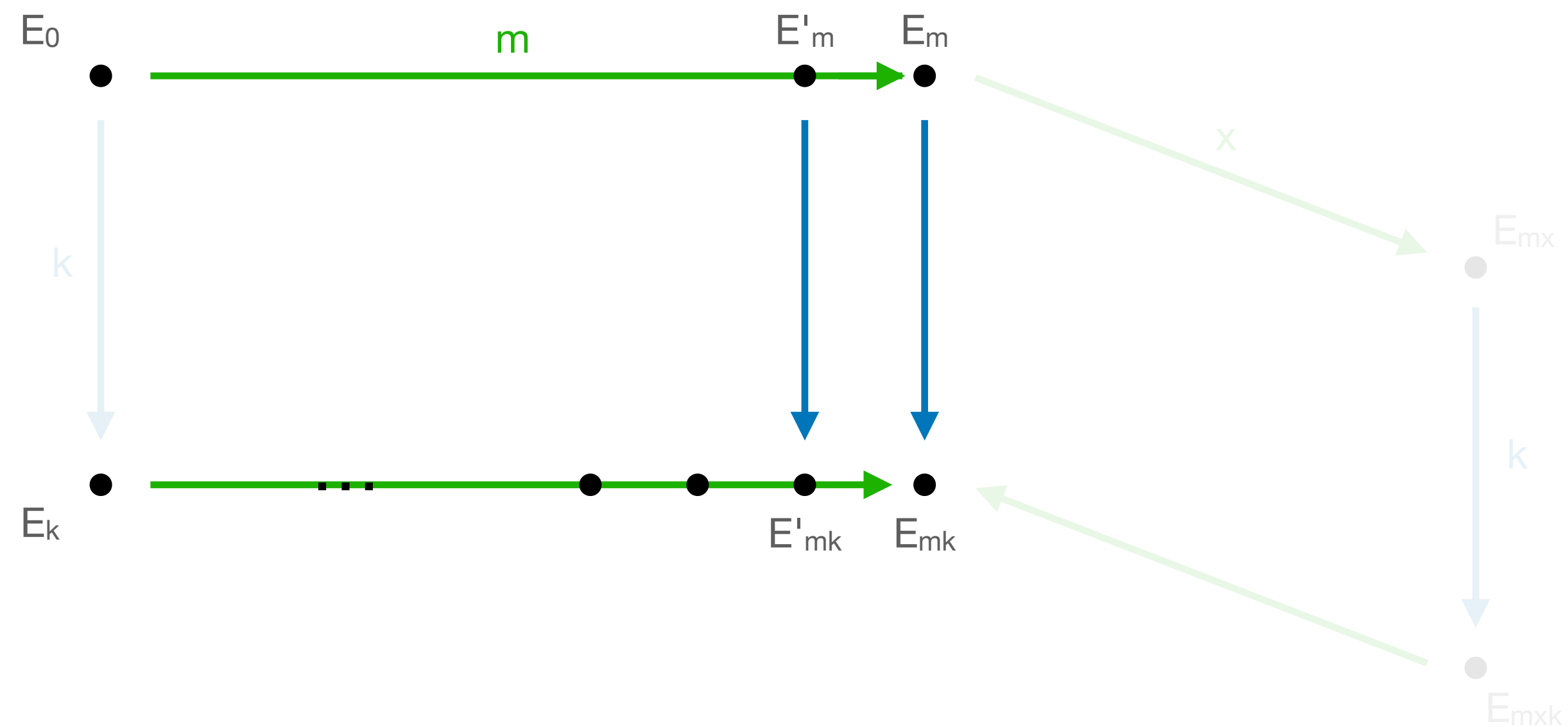
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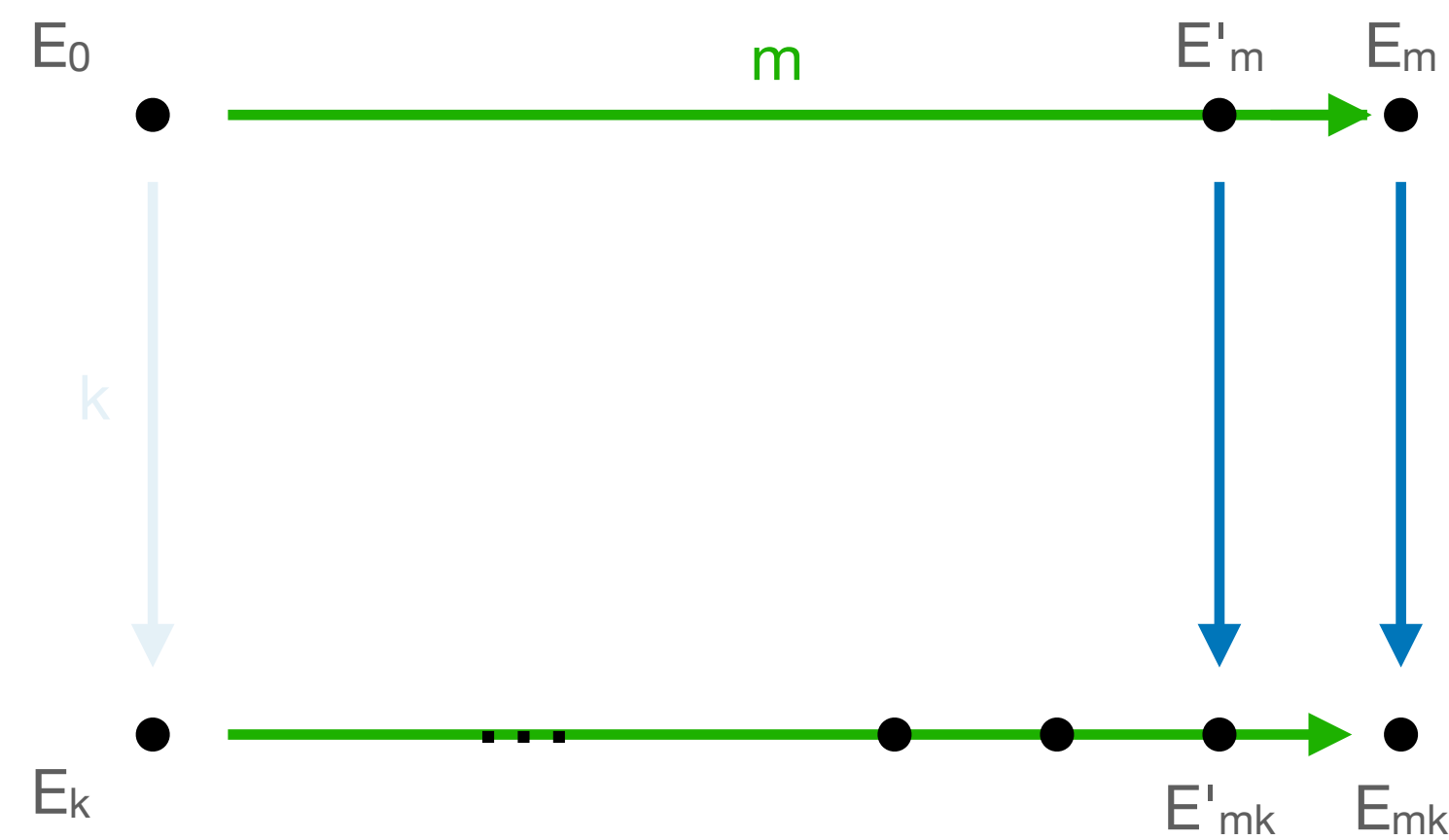
Part 1



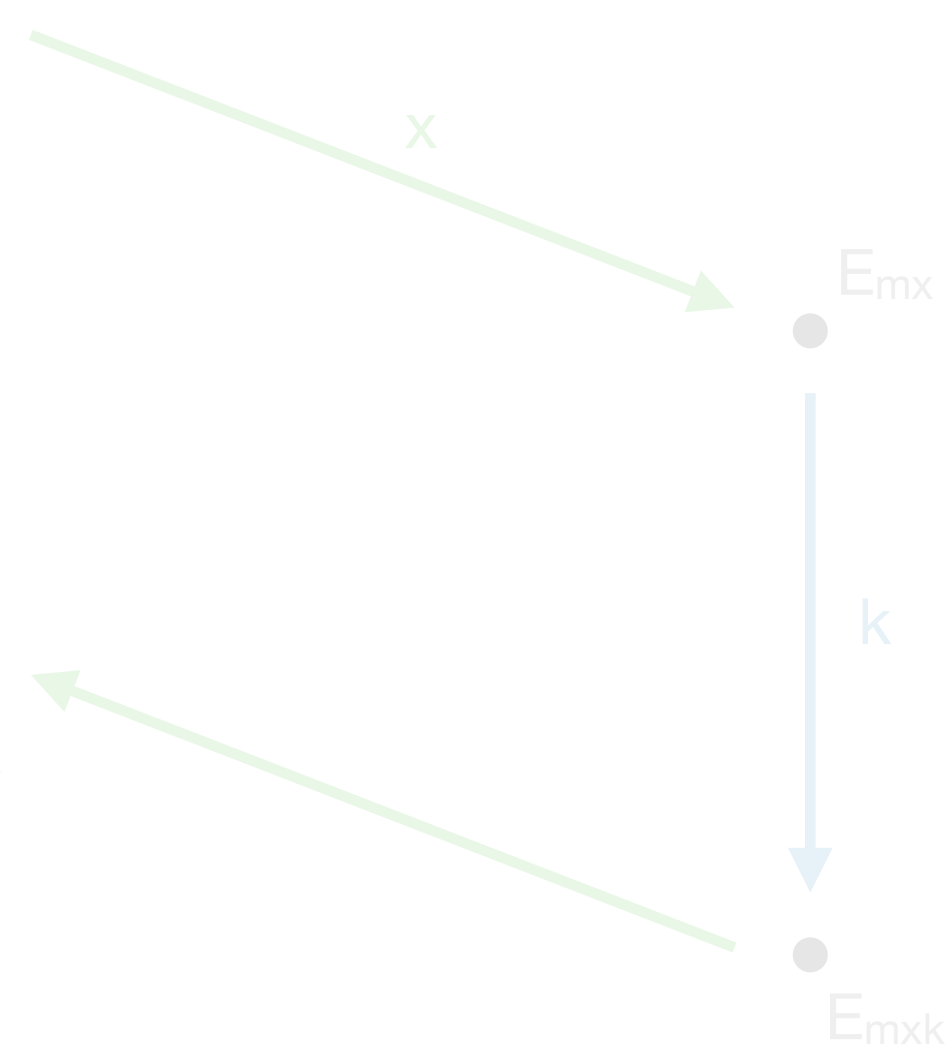
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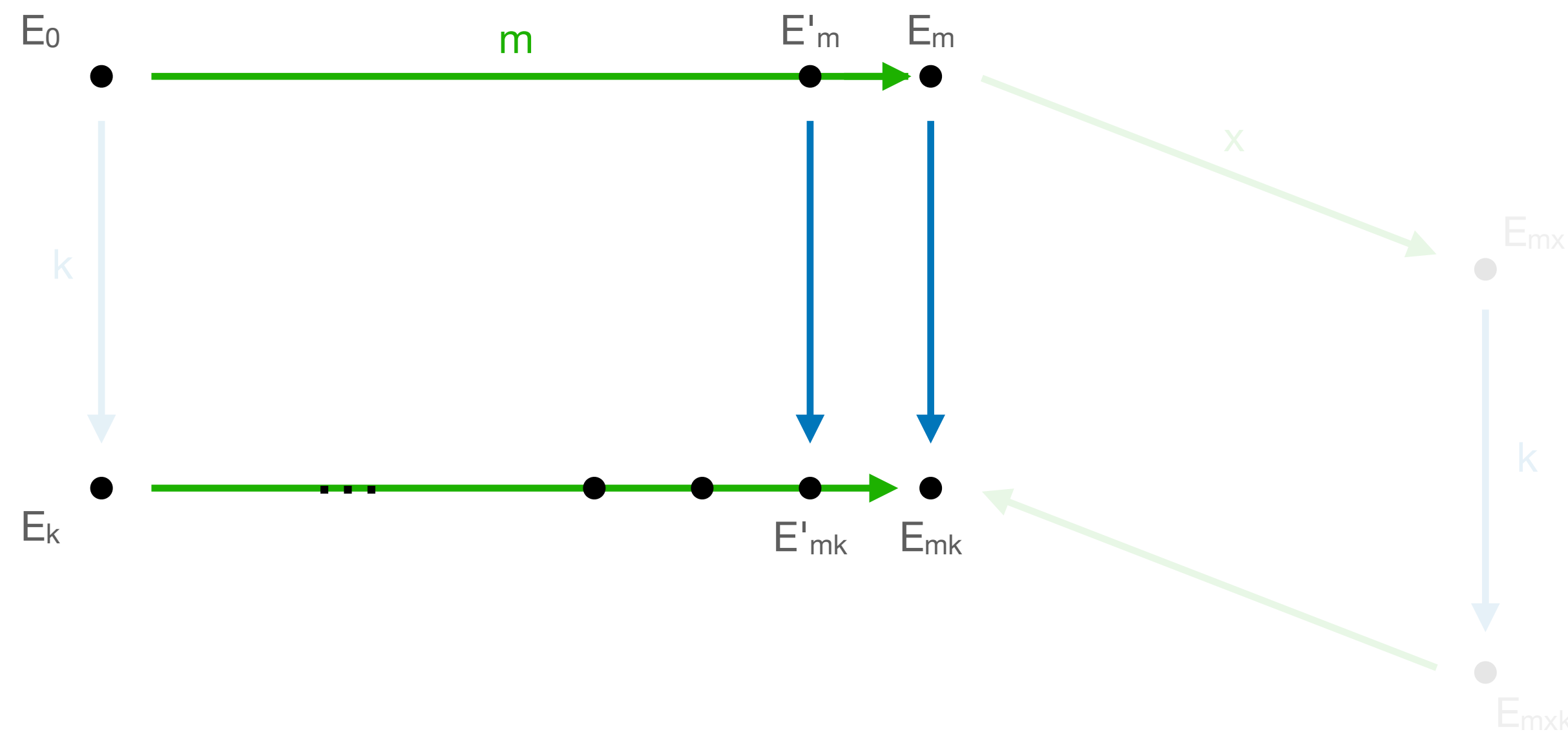
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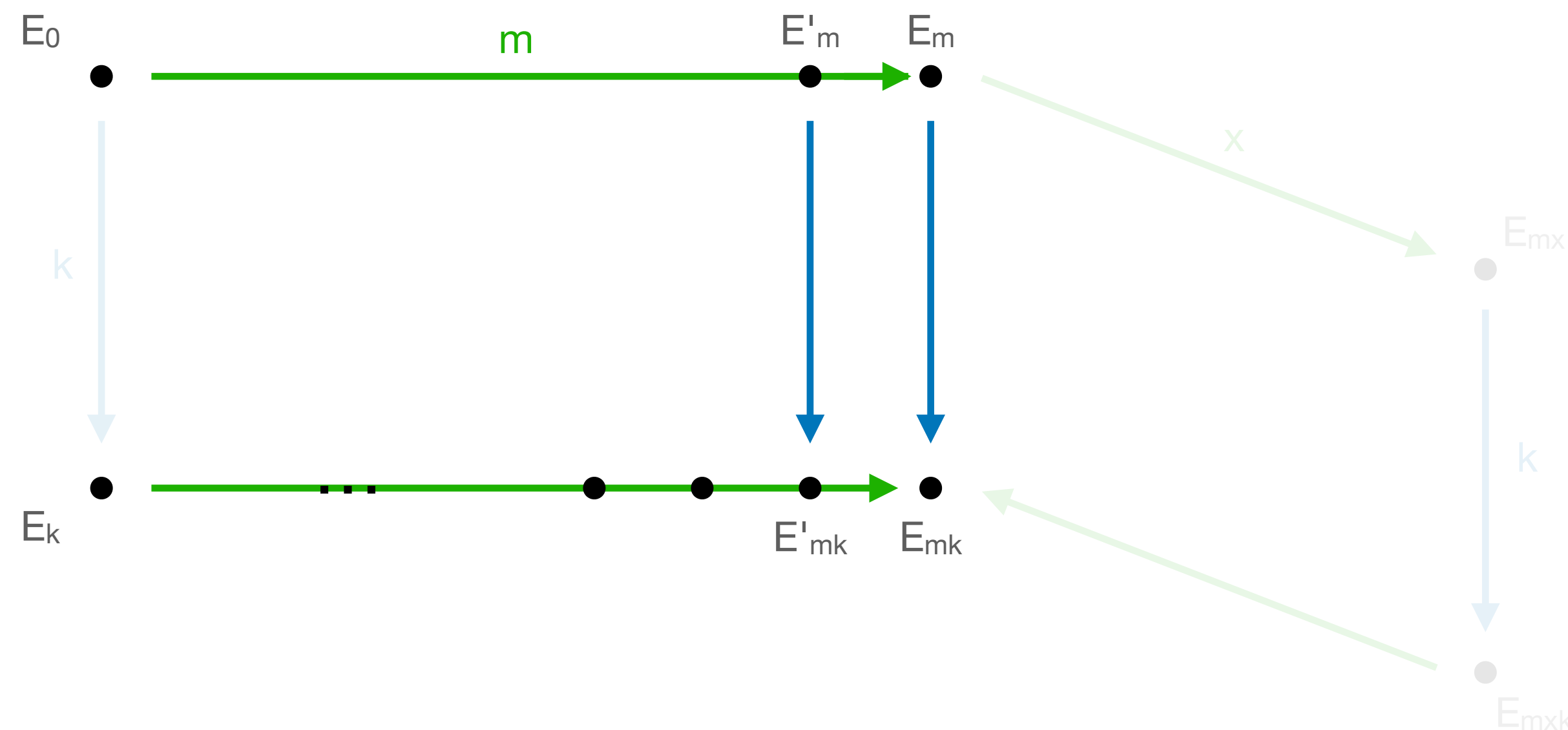
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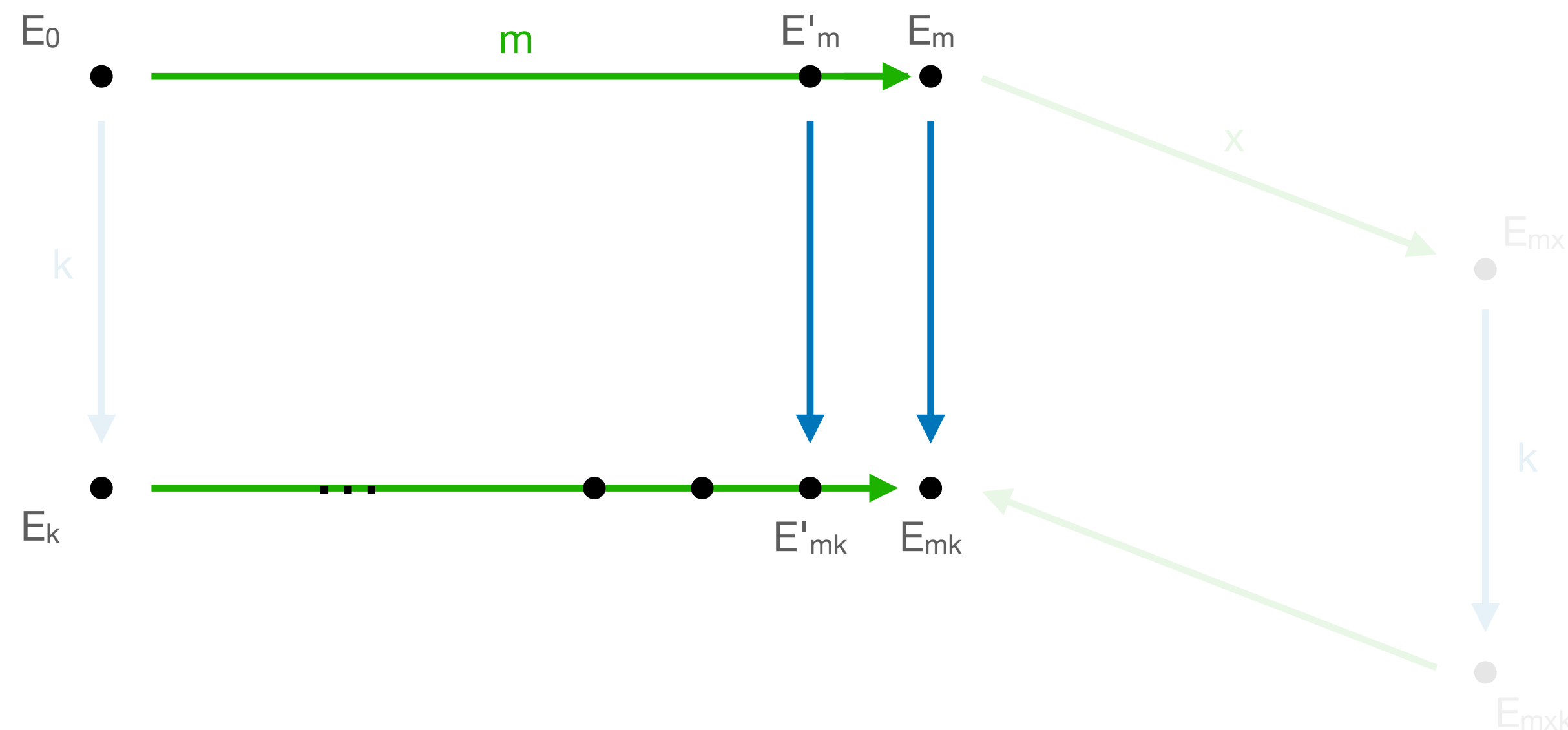
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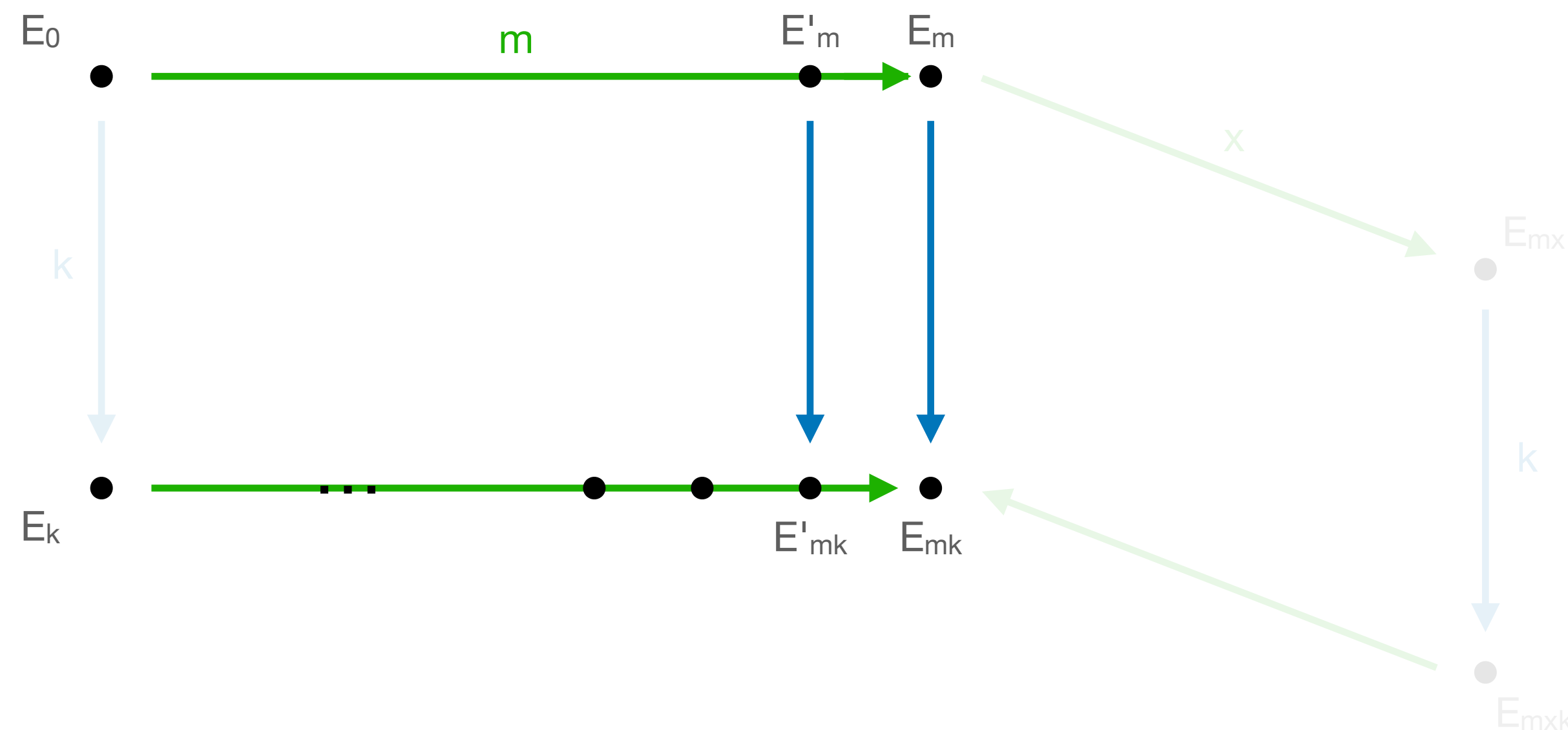
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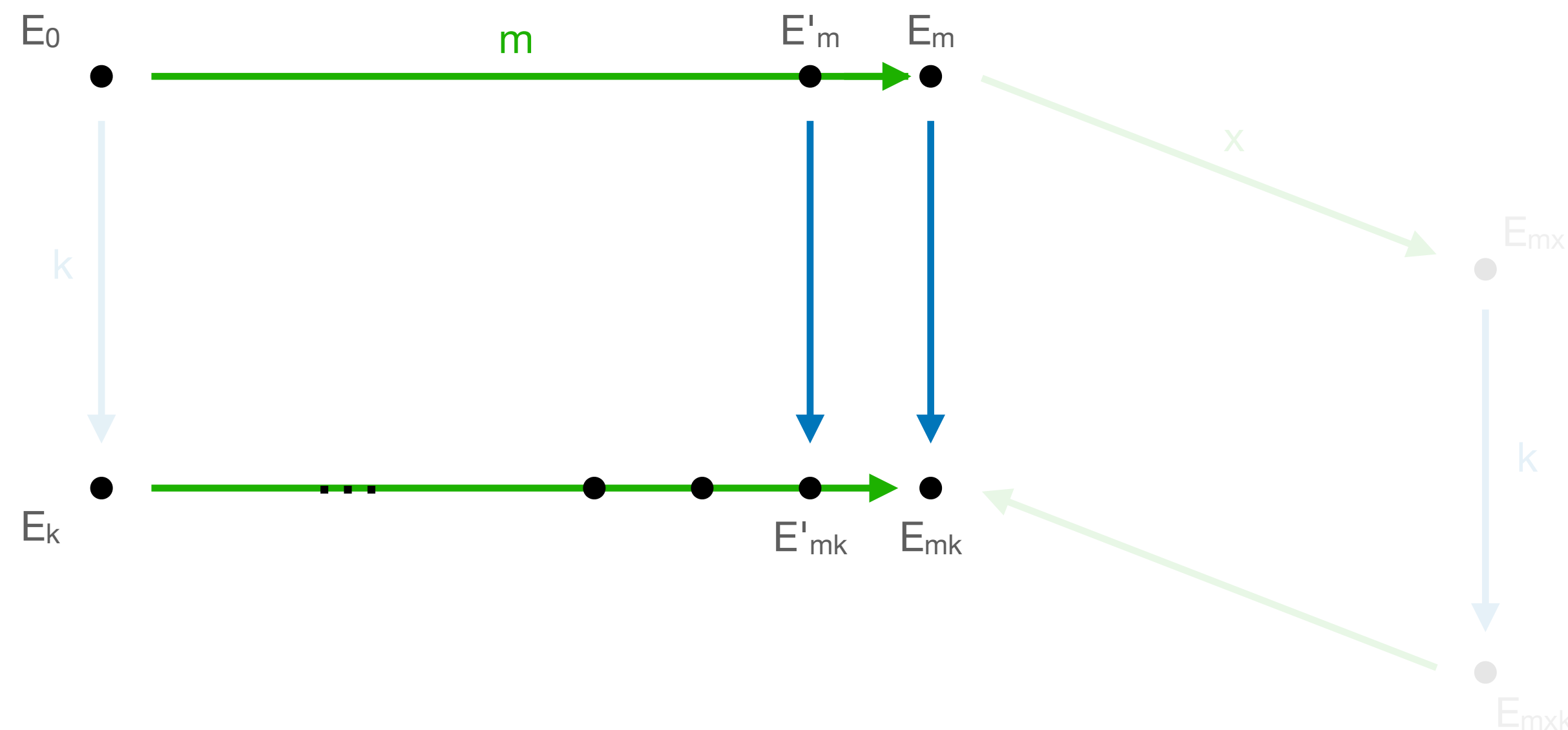
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The server can check the degree with the PoK!

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Part 2

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Actual complexity: sub-exponential

# Countermeasures?

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## **Validate more**

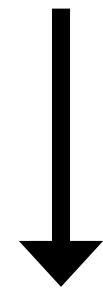
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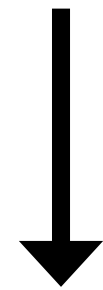
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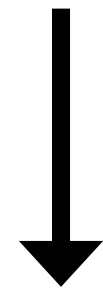
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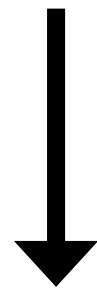
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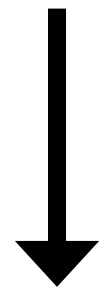
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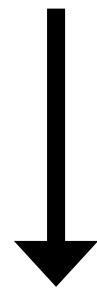
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## Scale parameters

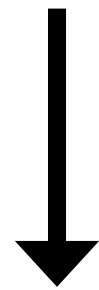
Attack is sub exponential

# Countermeasures?

It seems hard to prevent an attacker from recovering a basis on  $E_k$

## Validate more

Ensure that the client submits  
valid message isogenies



The protocol is oblivious

## Update values

Use dynamic values for  
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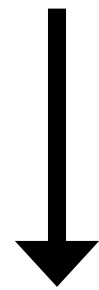
$p > 2^{16,000}$

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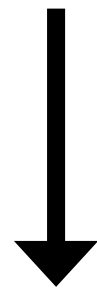
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**Idea:** make the basis on  $E_k$  not enough for an attack

# An efficient countermeasure

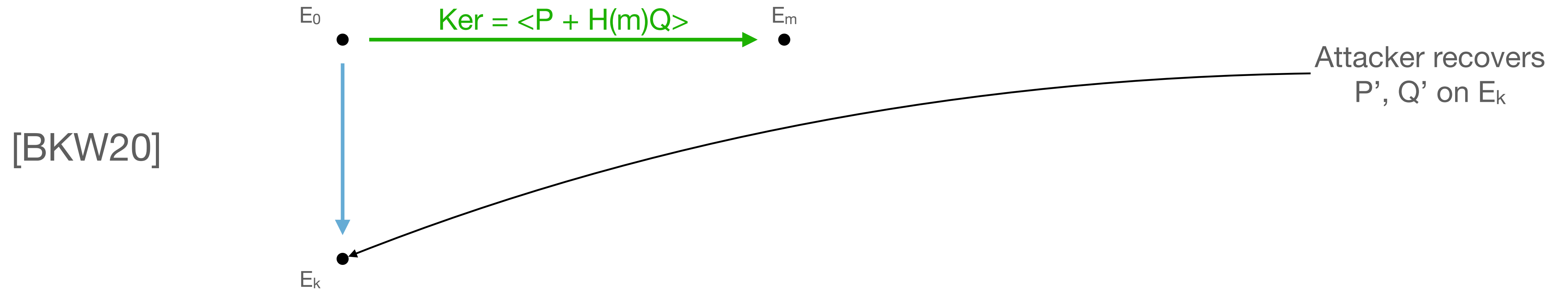
[BKW20]

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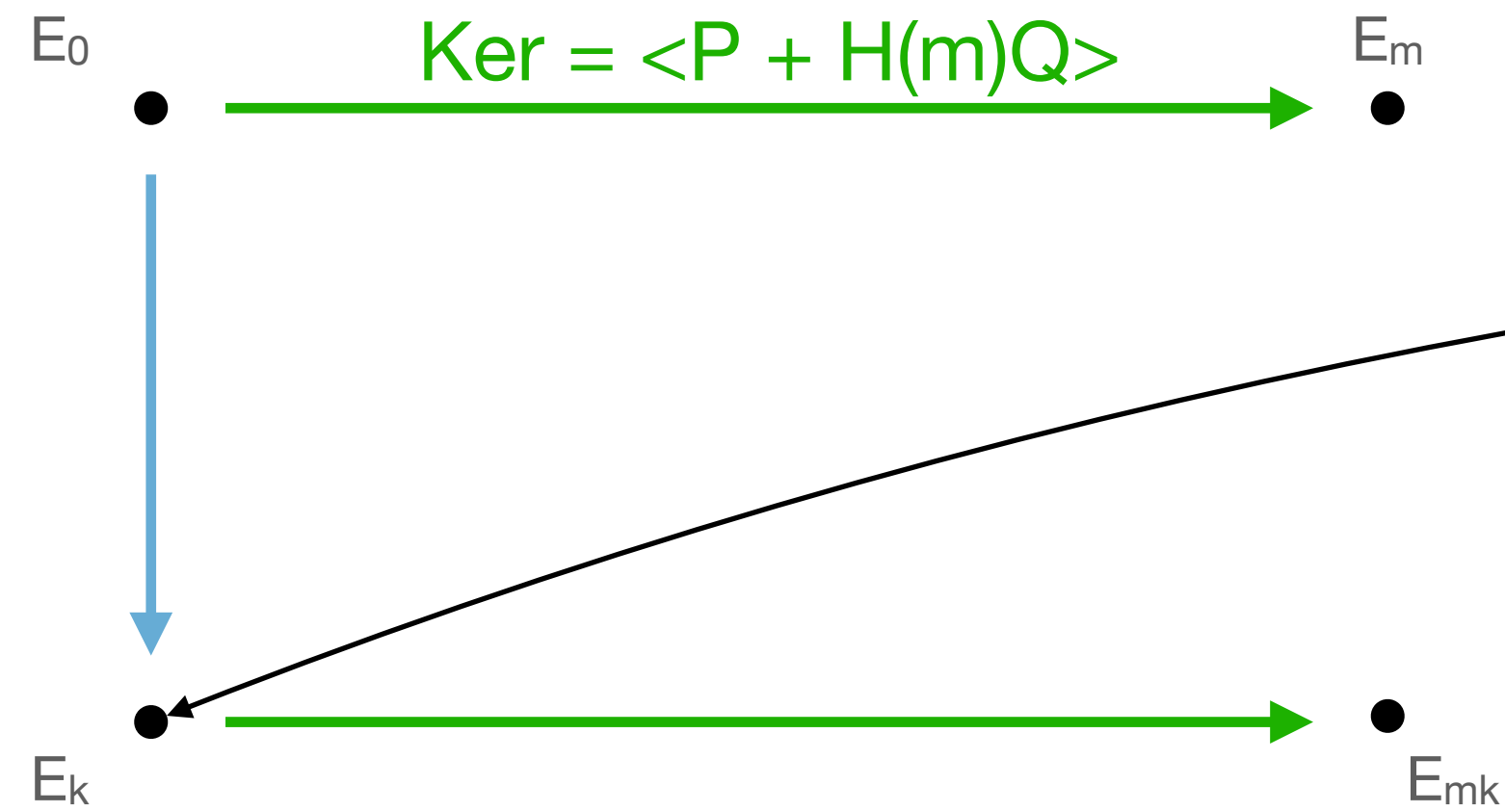
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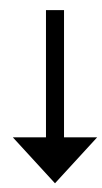


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[BKW20]

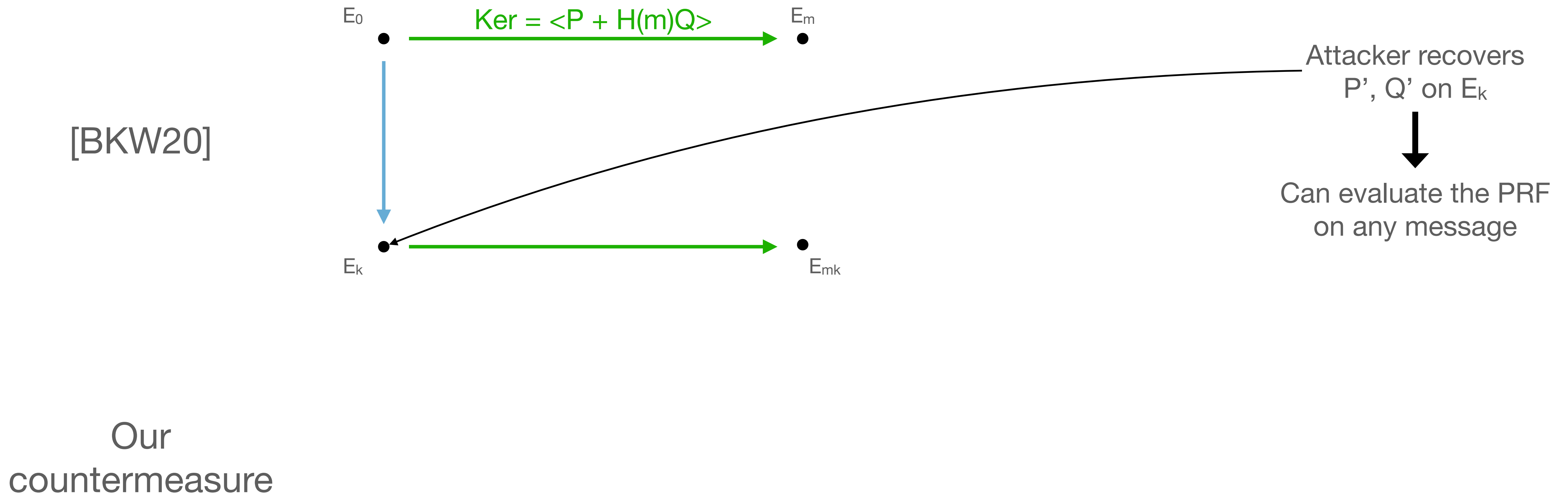


Attacker recovers  
 $P', Q'$  on  $E_k$

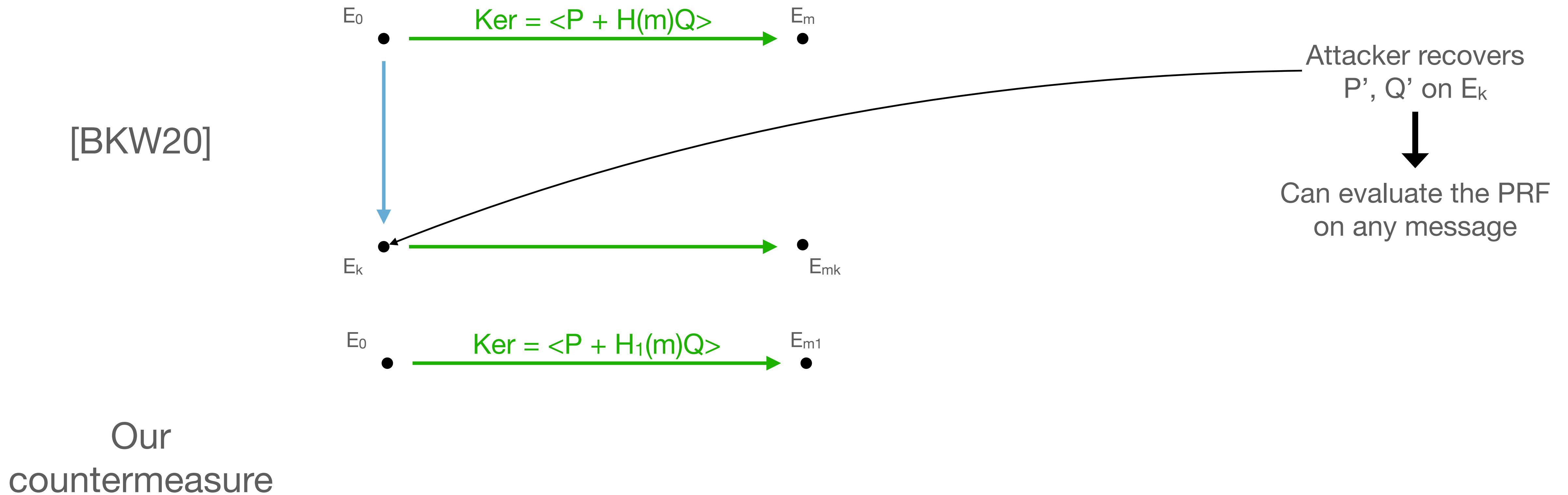


Can evaluate the PRF  
on any message

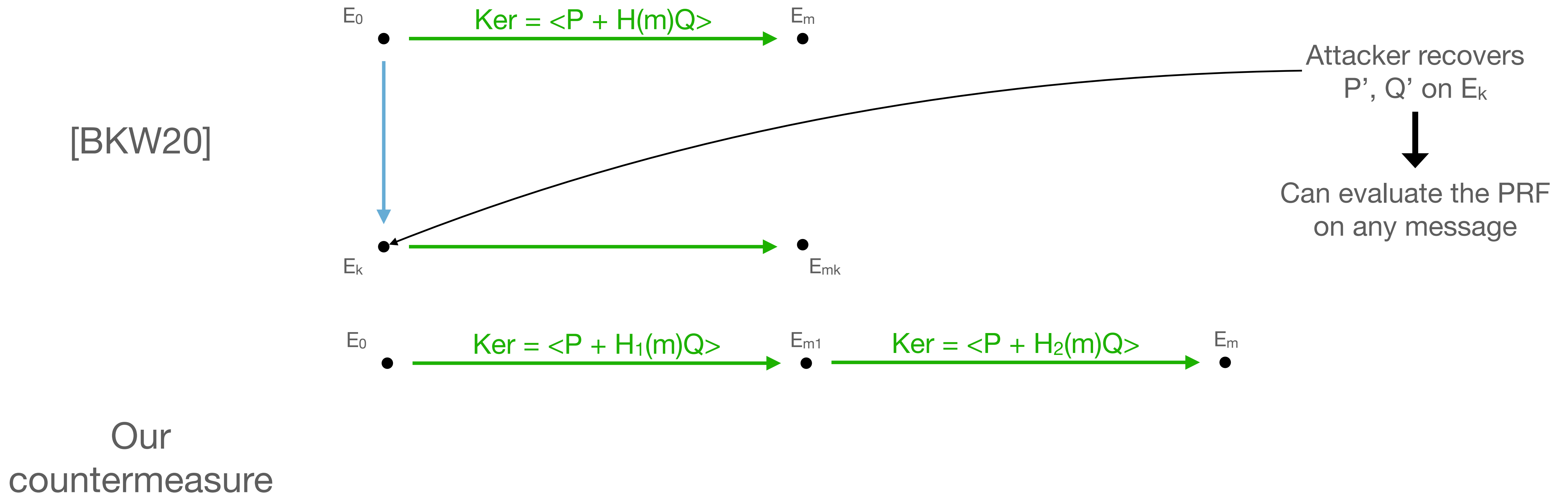
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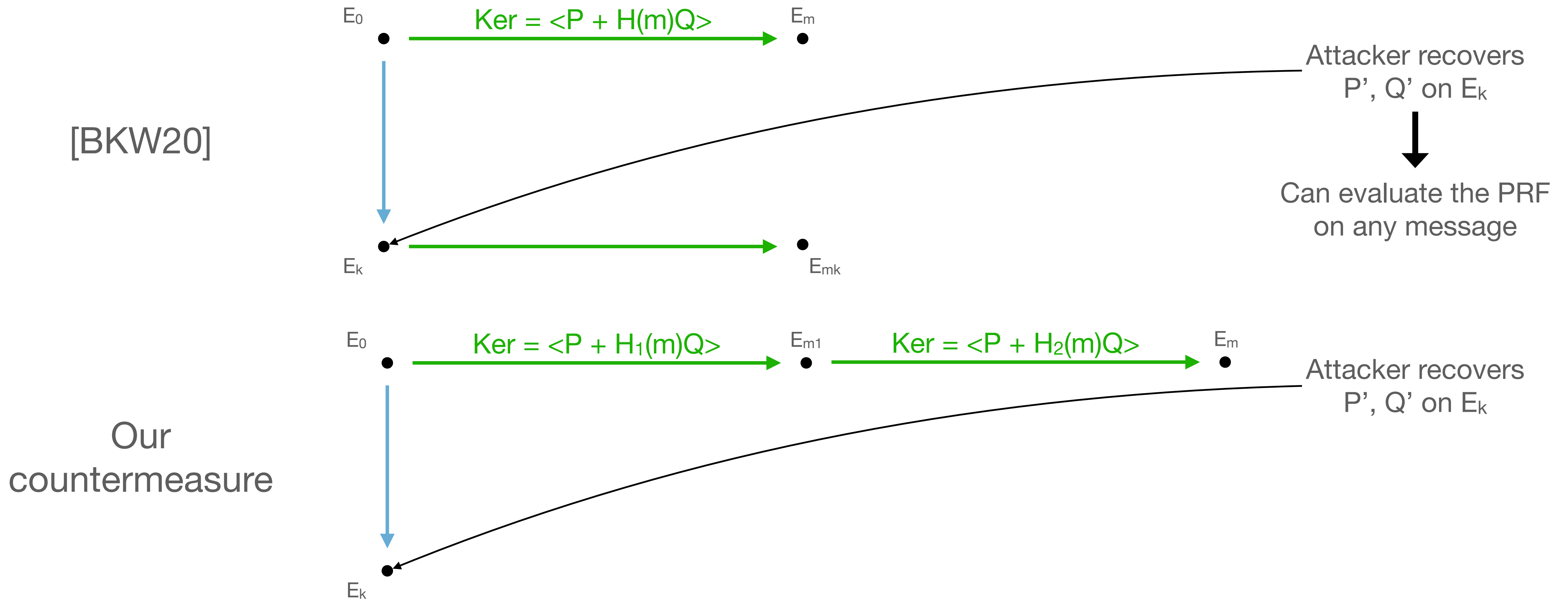
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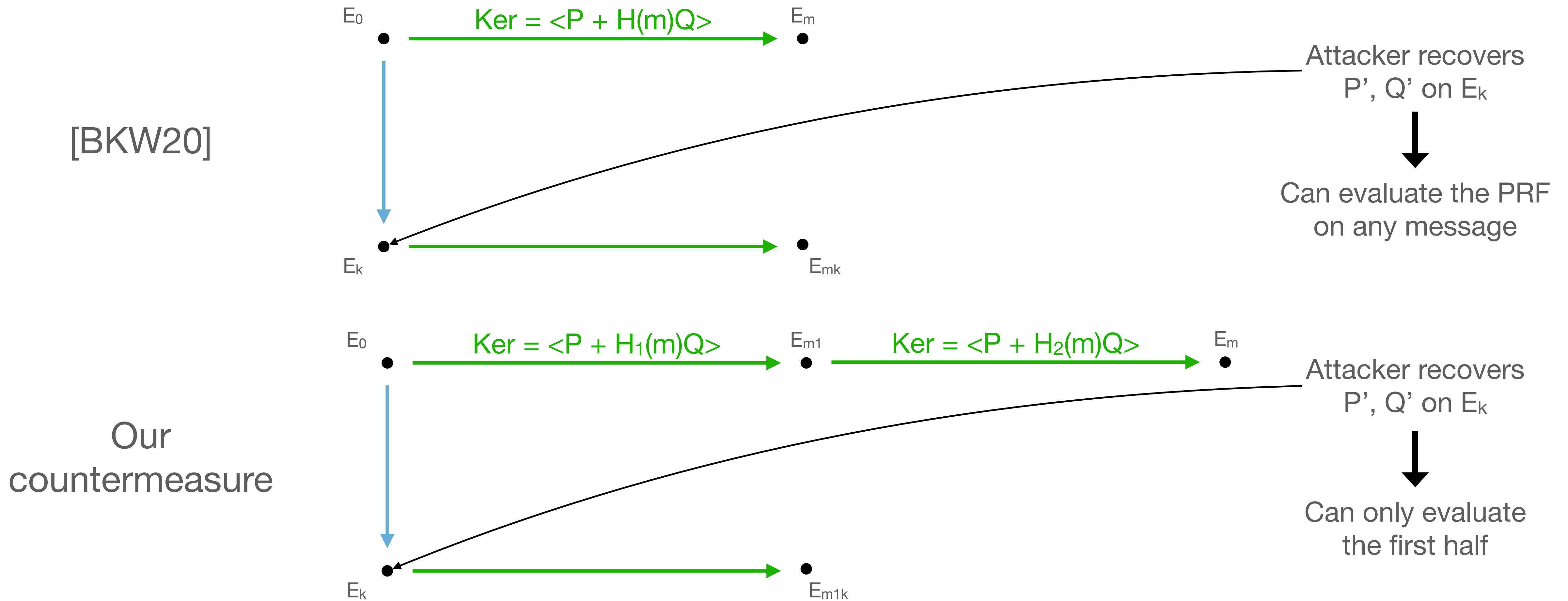
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$p \approx 26000$

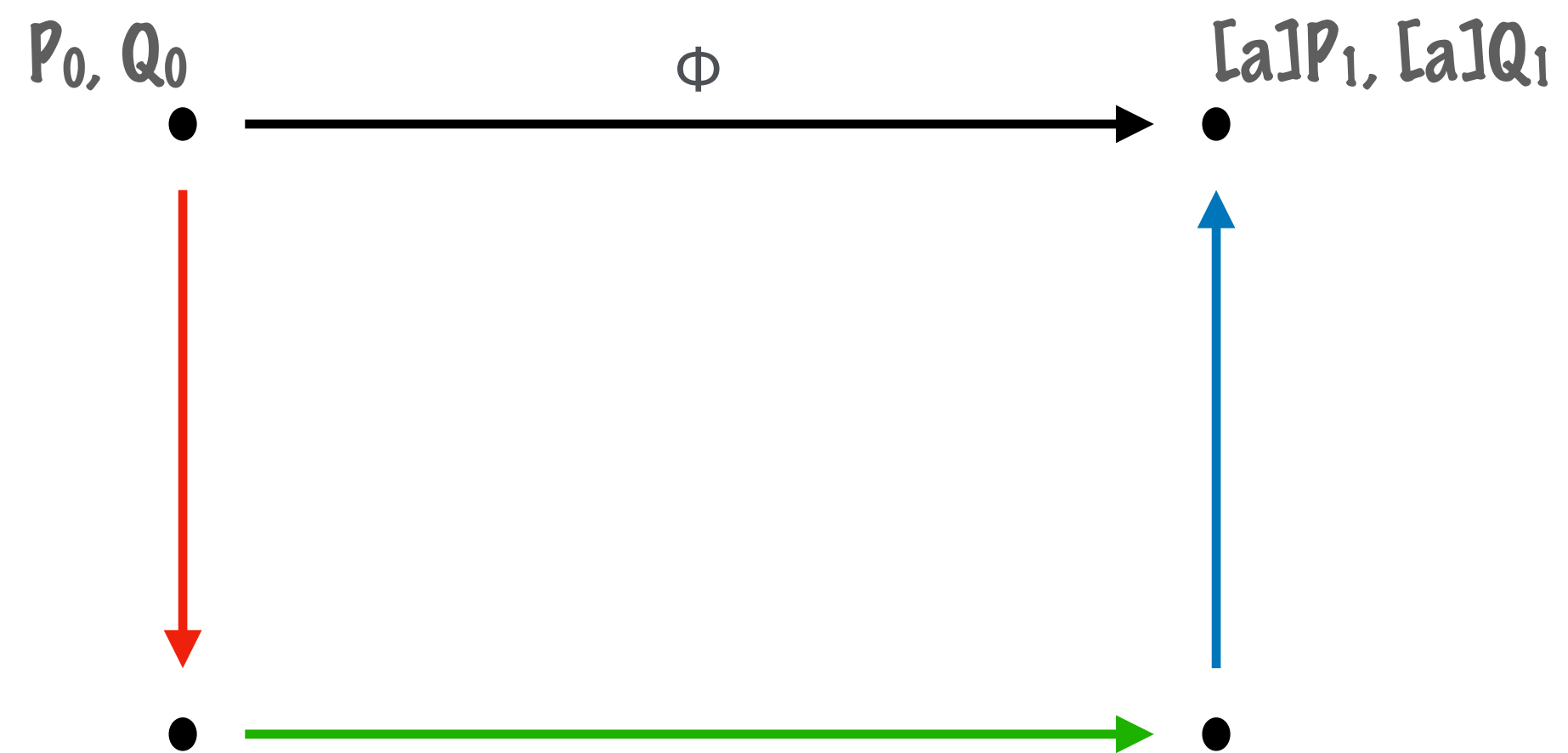
# PolK with masked torsion



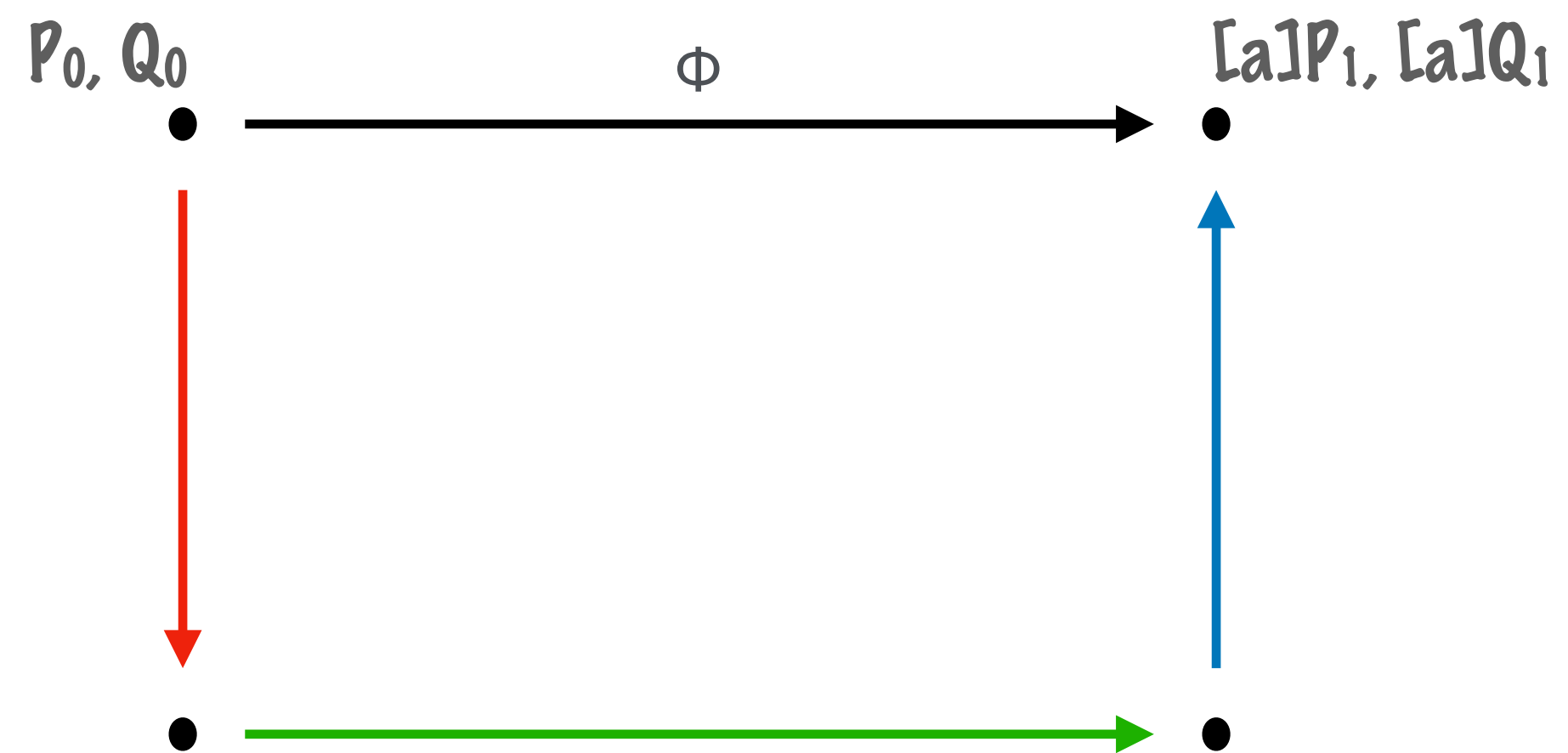
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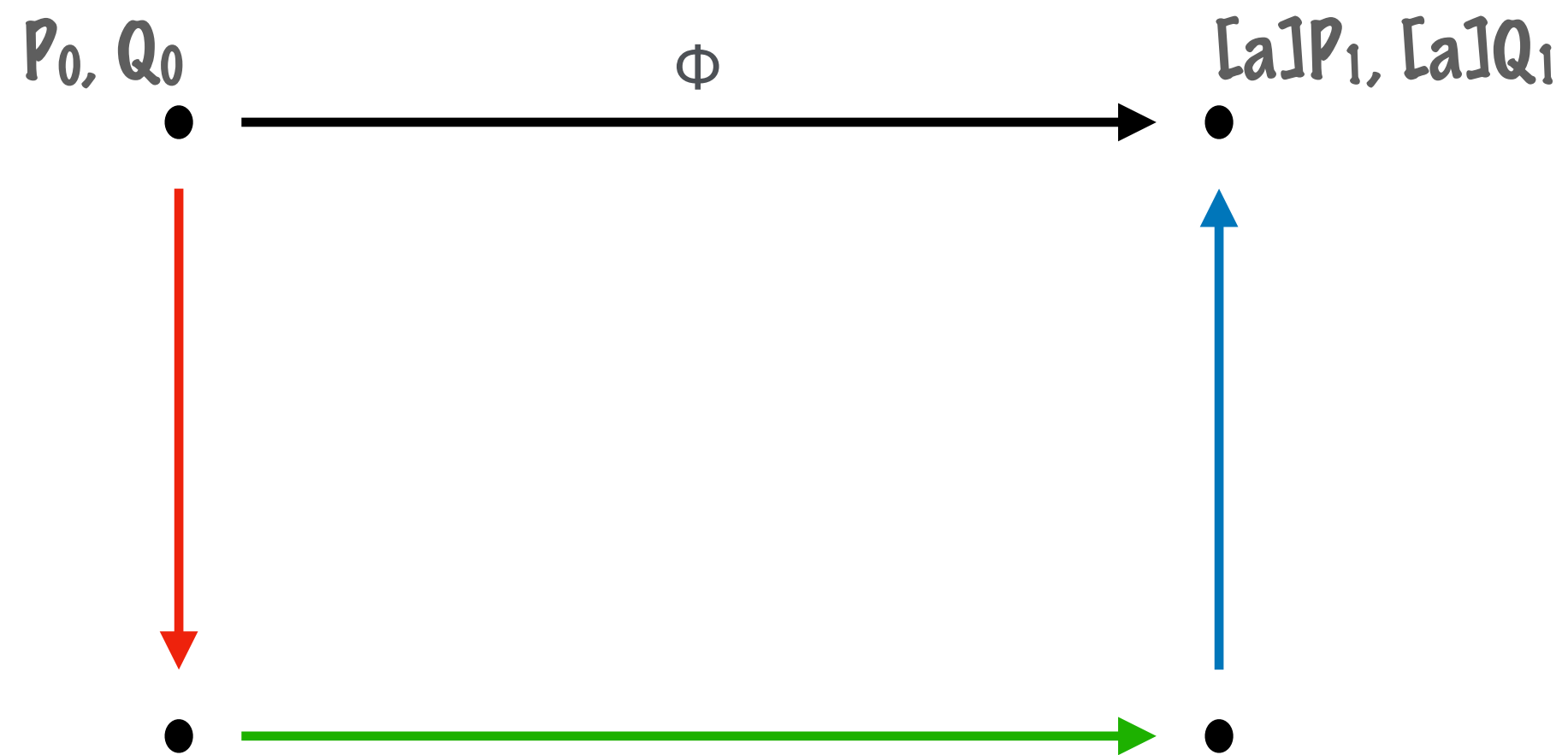


# PolK with masked torsion



challenges from  $\{-1, 0, 1\}$

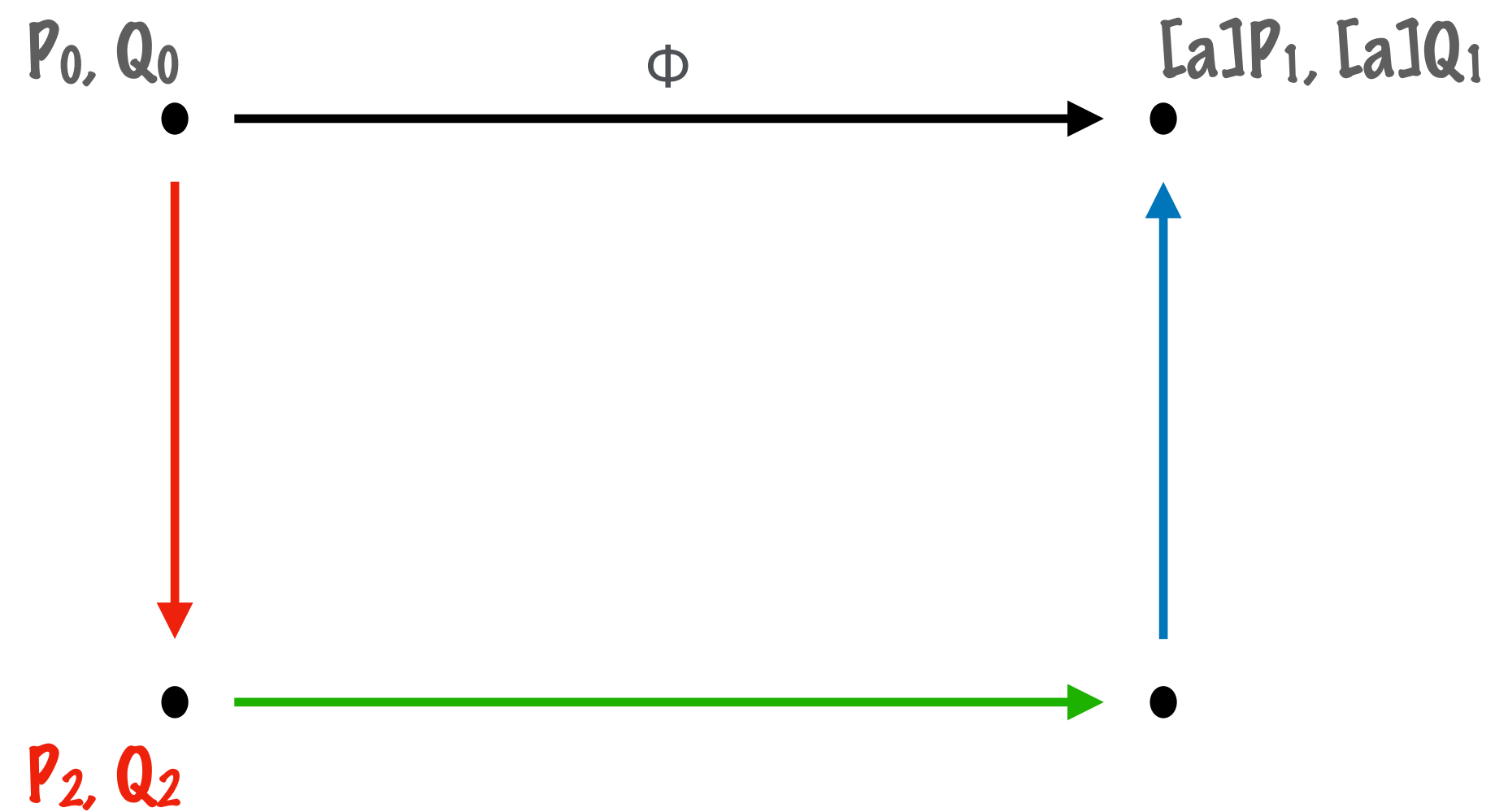
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 $\Rightarrow$  need  $1.7\lambda$  repetitions

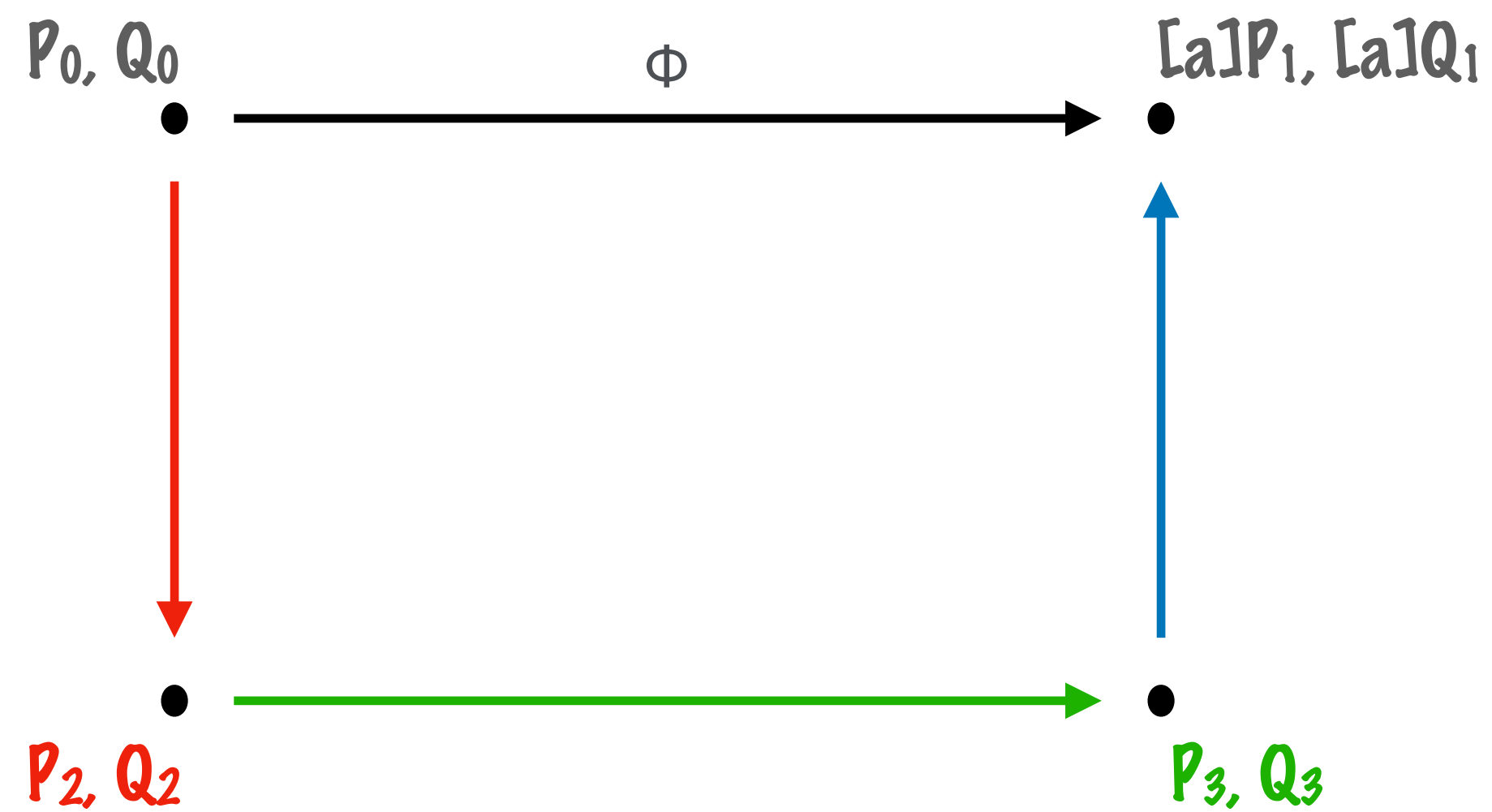
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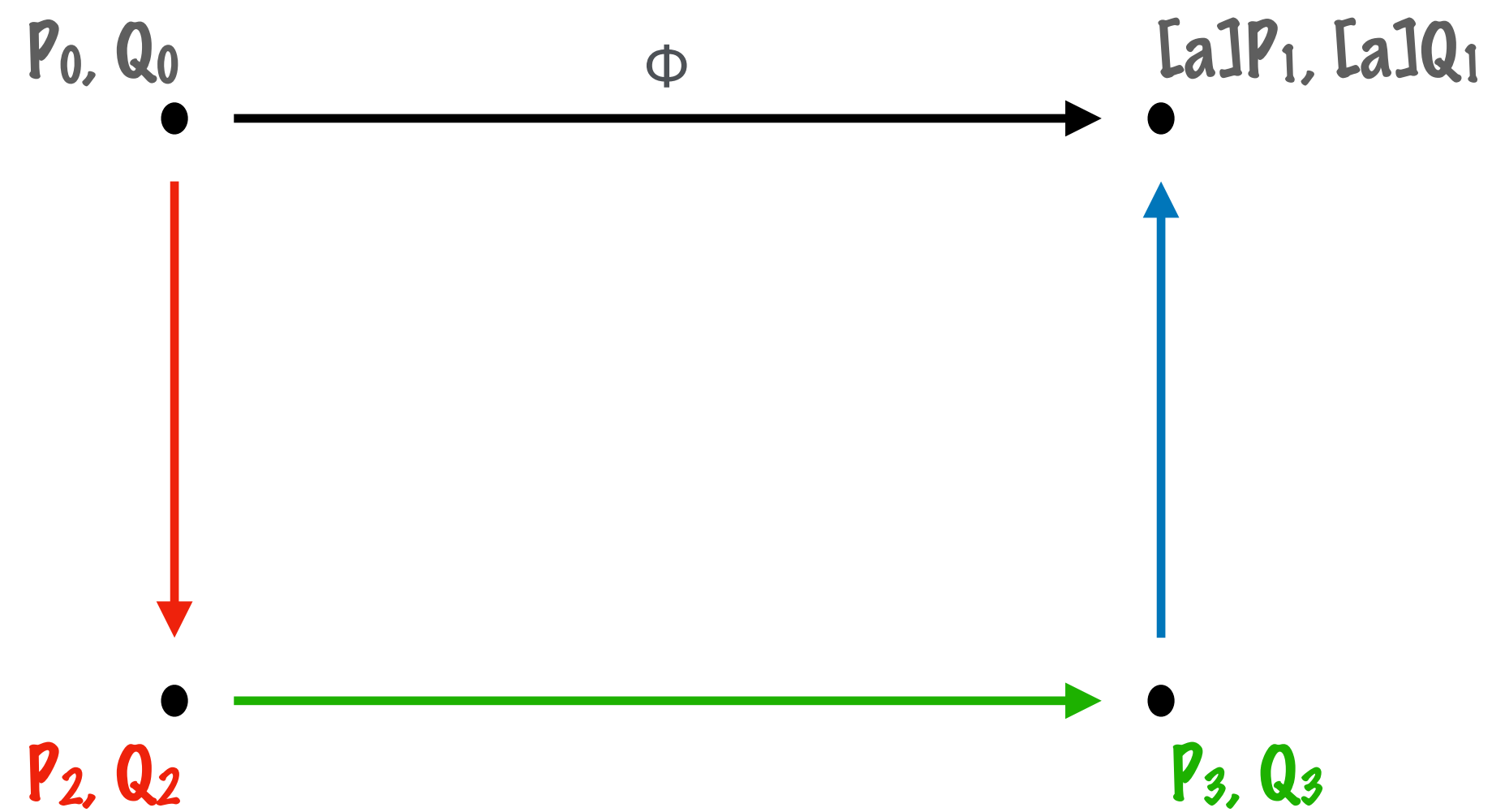
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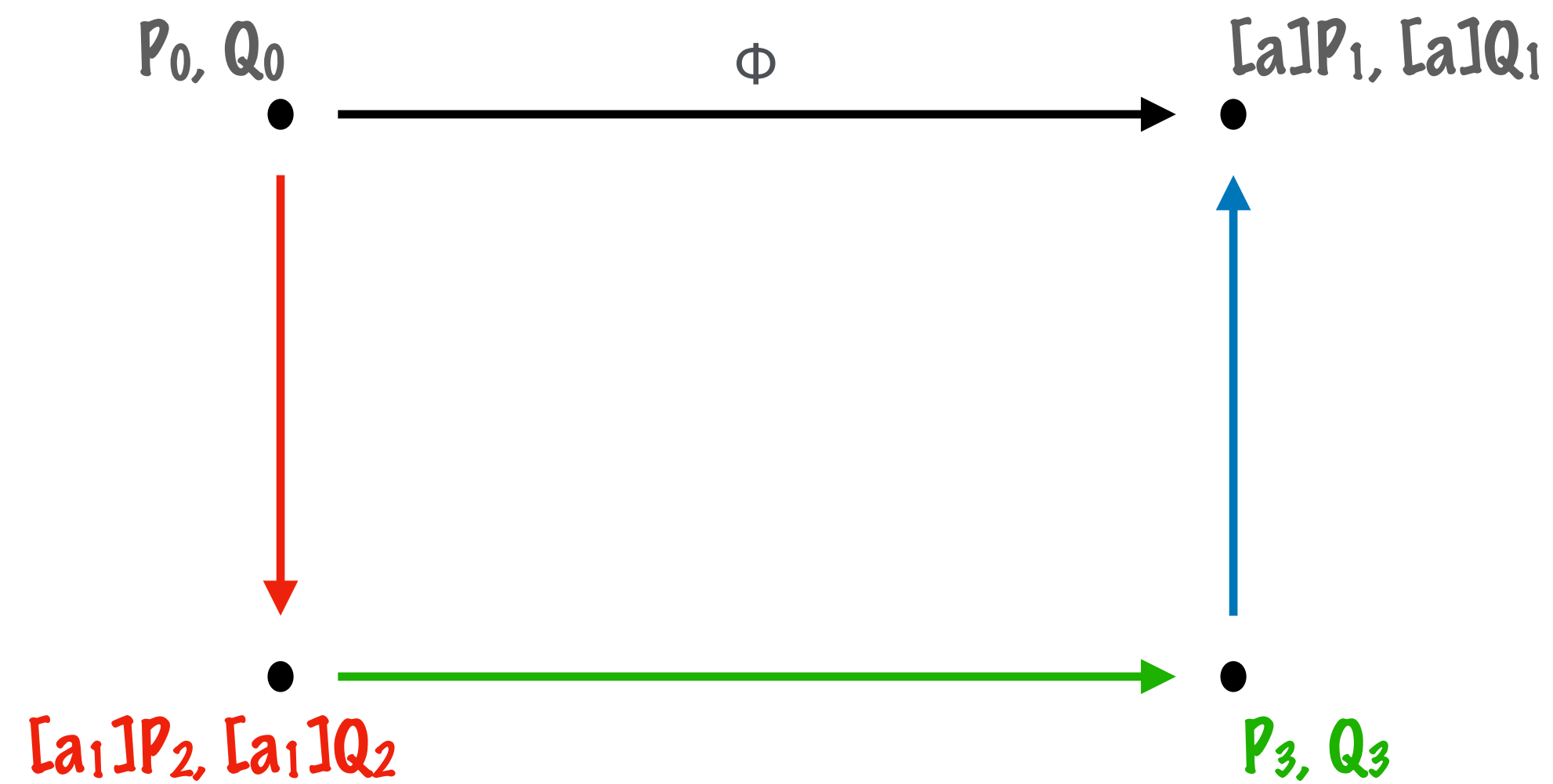


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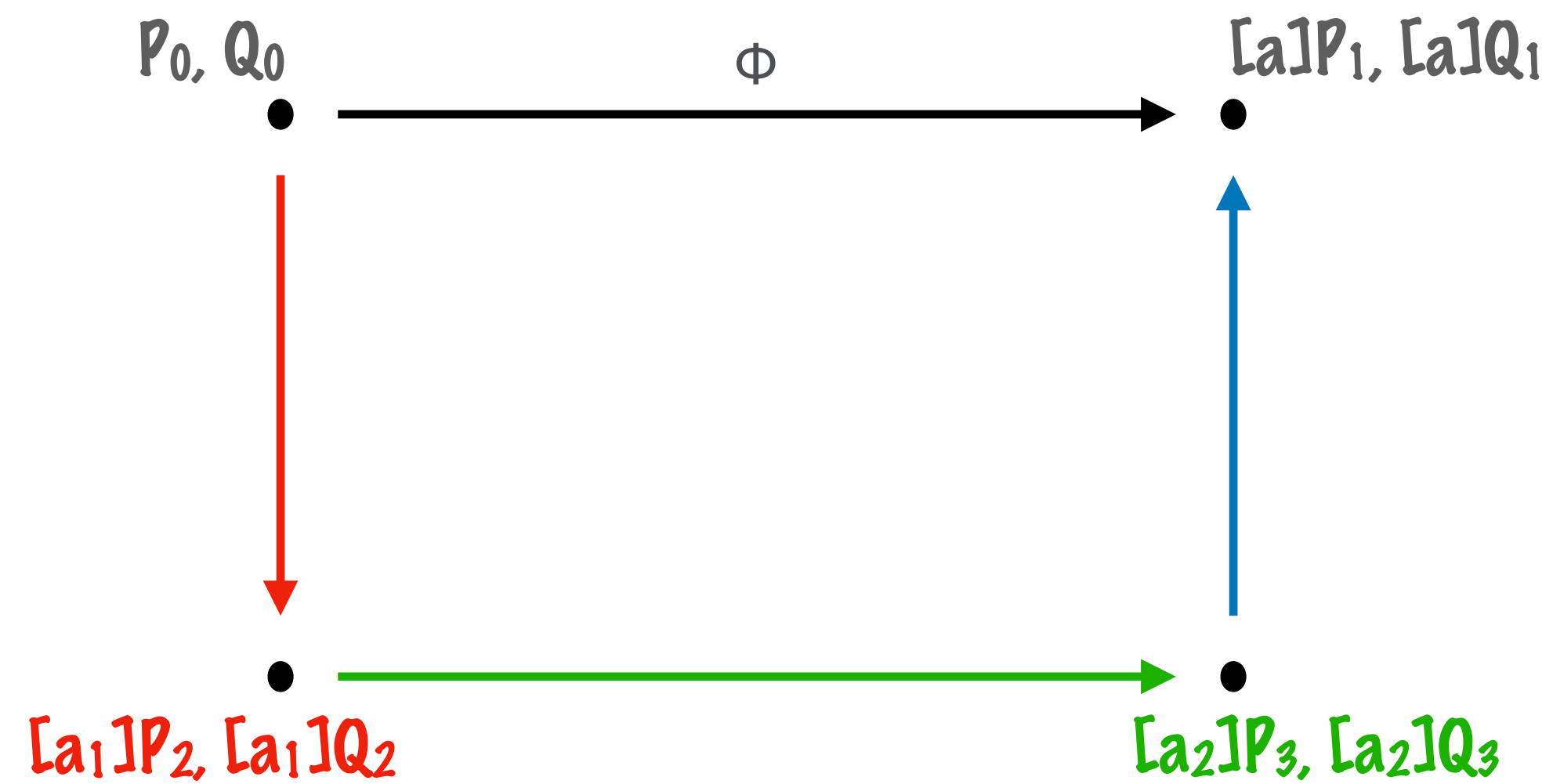


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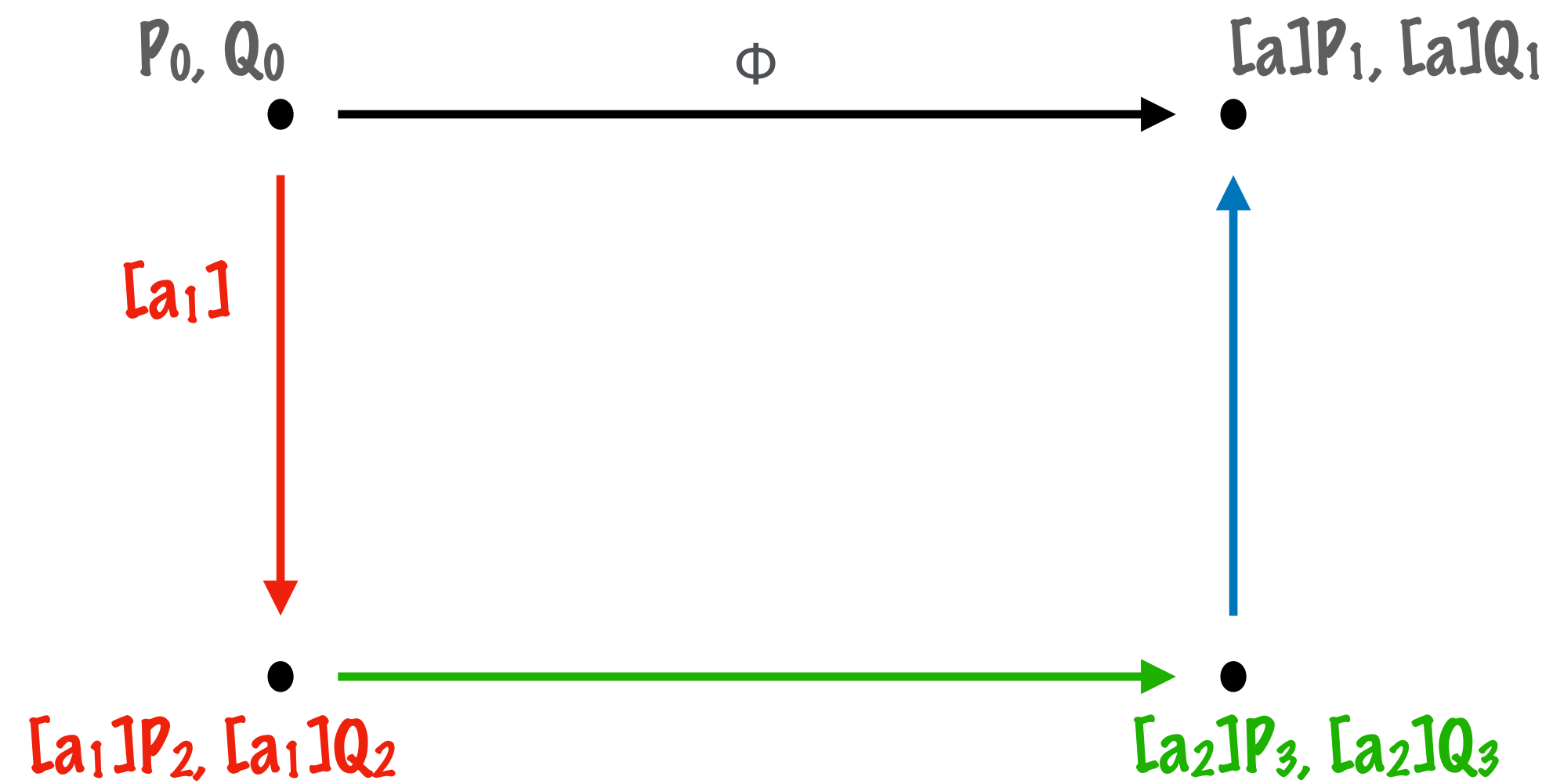


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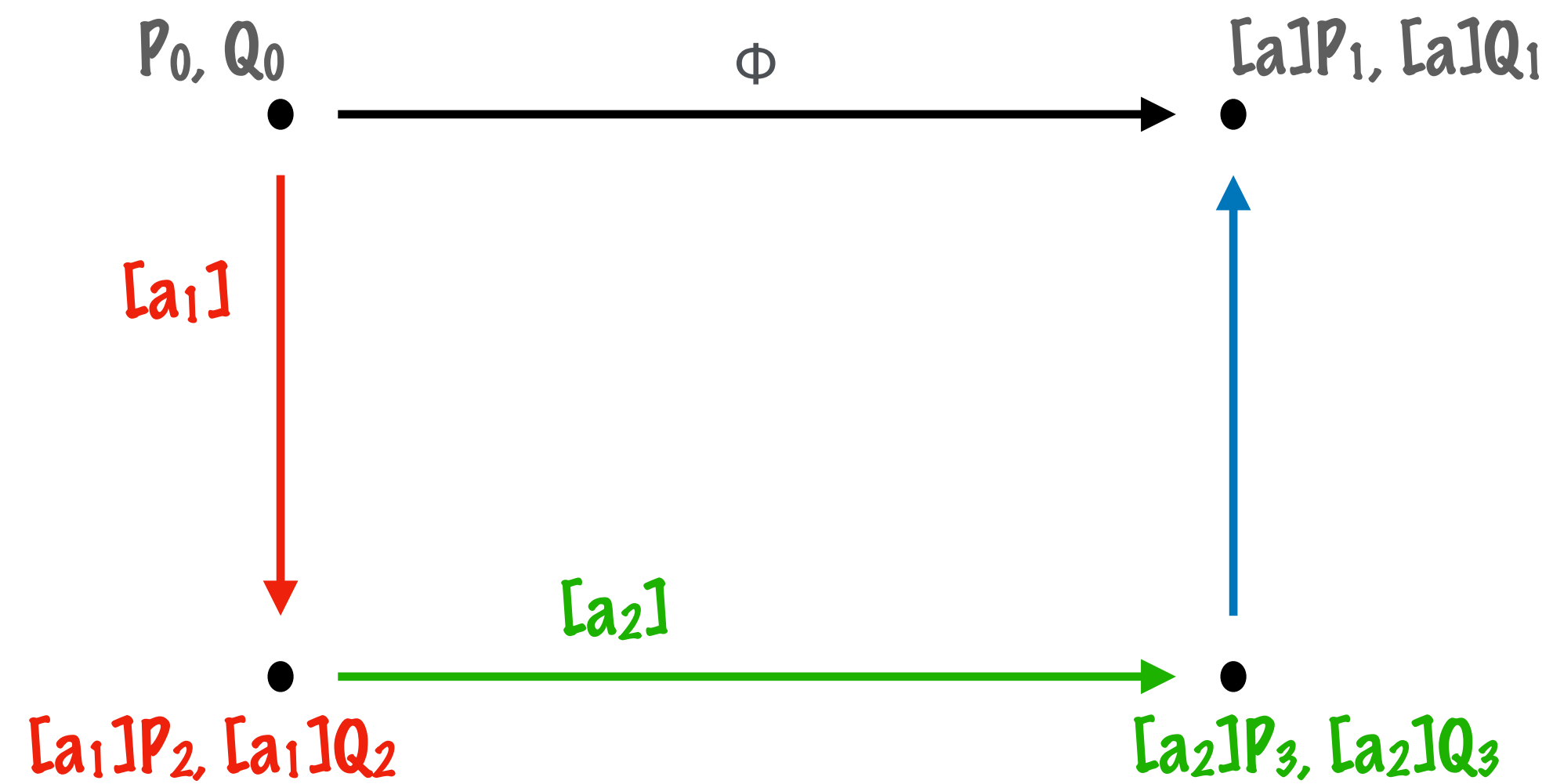


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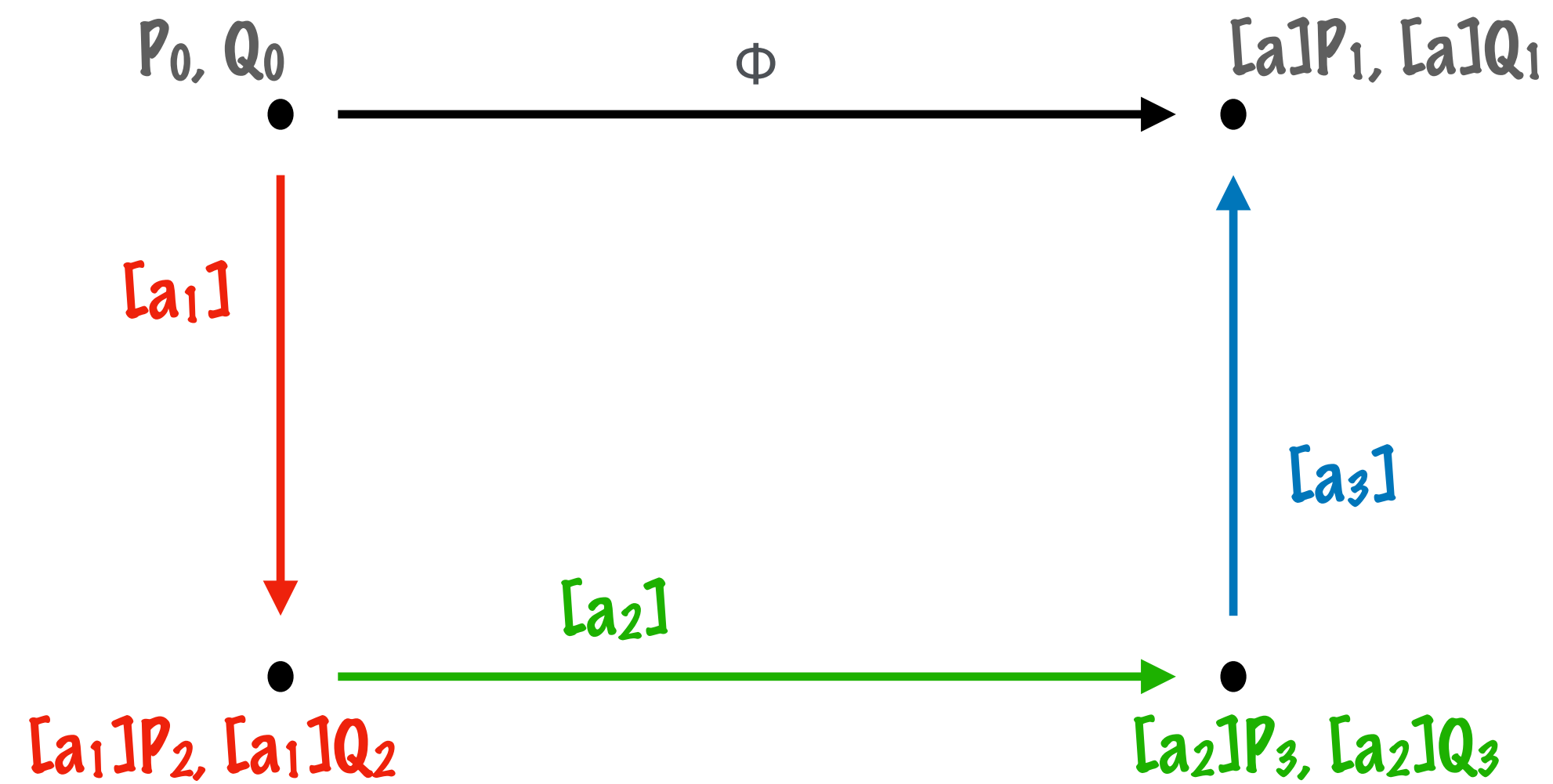


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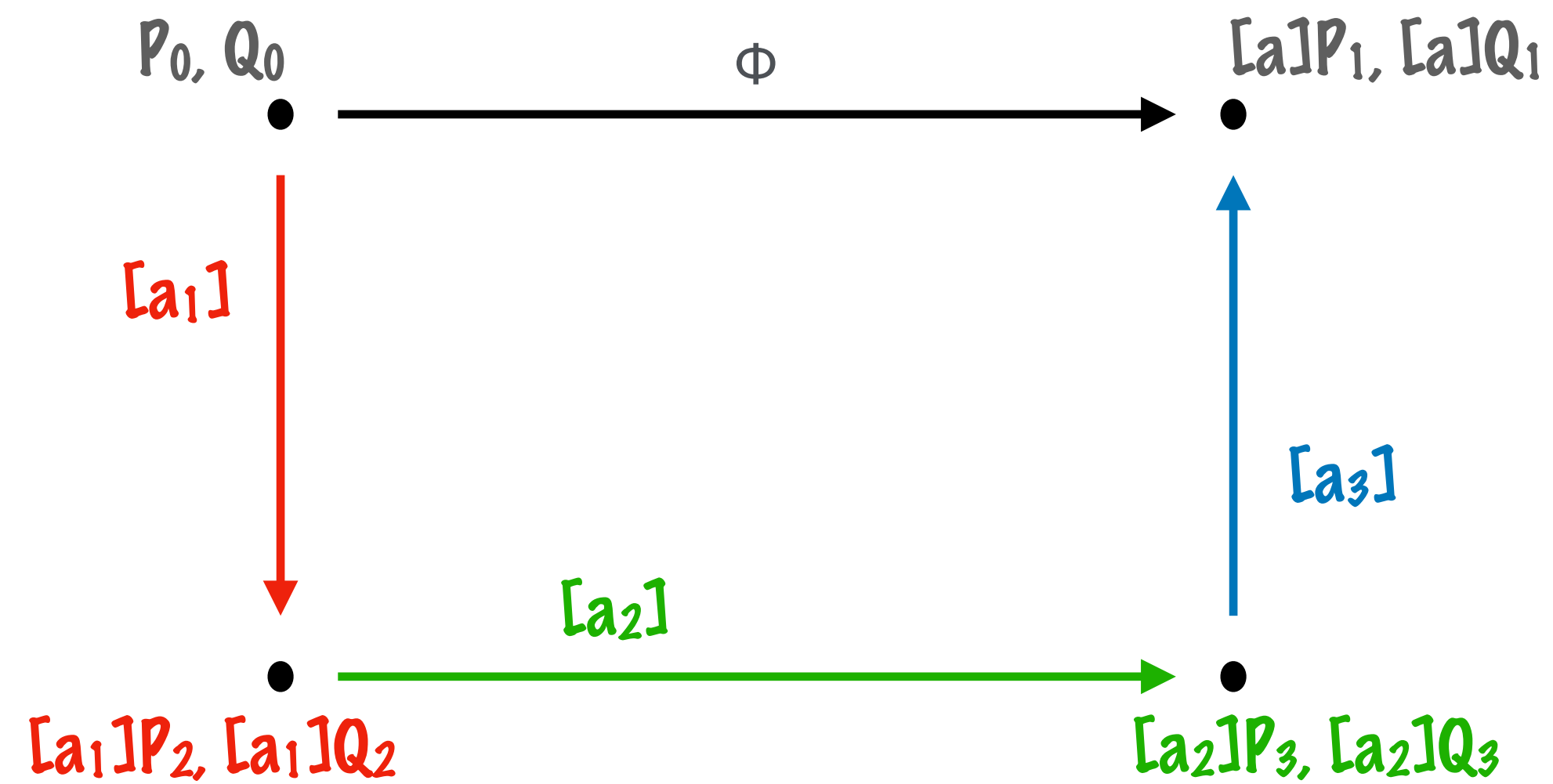


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$$p \approx \text{ord } P, Q \times \deg \Phi \times \deg \rightarrow \approx 2^{9000}$$

# Verifiability

[BKW20] uses 3 proofs:



Server's isogeny



Server's commitment



Isogeny is parallel  
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Prove "parallelness" when  
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*Non-interactive*



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[BKW20] uses 3 proofs:



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Prove "parallelness" when  
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*Non-interactive*  
*Saves computations*



Isogeny is parallel  
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**Interactive (5 rounds)**

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- New PoPI **more efficient than original**  
**round optimal**

# Results

Protocol	Rounds	Bandwidth (avg.)	Verifiable	Secure
[1] (LWE)	2	>128 GB	✓	✓
[5] (CSIDH)	3	424 kB	✗	✓
[5] (SIDH) <sup>FO</sup>	6	1.4 MB	✓	✗
[5] (SIDH) <sup>Unruh</sup>	6	>10.9 MB	✓	✗
[This work] <sup>FO</sup>	2	1.9 MB	✓	✓
[This work] <sup>Unruh</sup>	2	8.7 MB	✓	✓