

# A Post-Quantum Round-Optimal Oblivious PRF from Isogenies

**Andrea Basso**

16<sup>th</sup> August, 2023  
Selected Areas in Cryptography 2023

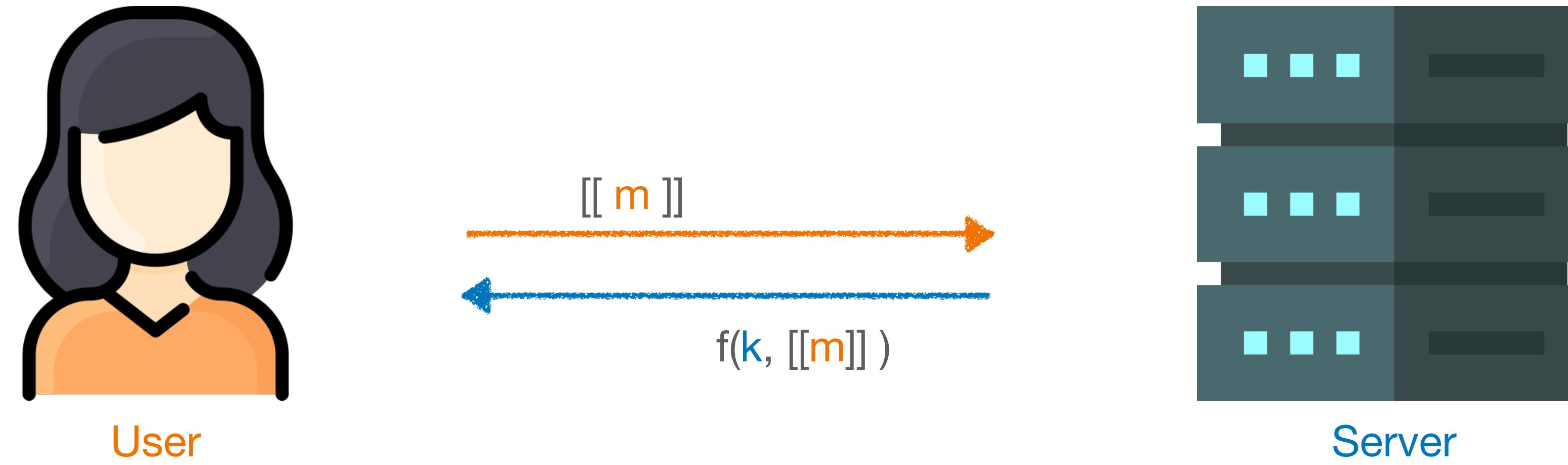


UNIVERSITY OF  
BIRMINGHAM

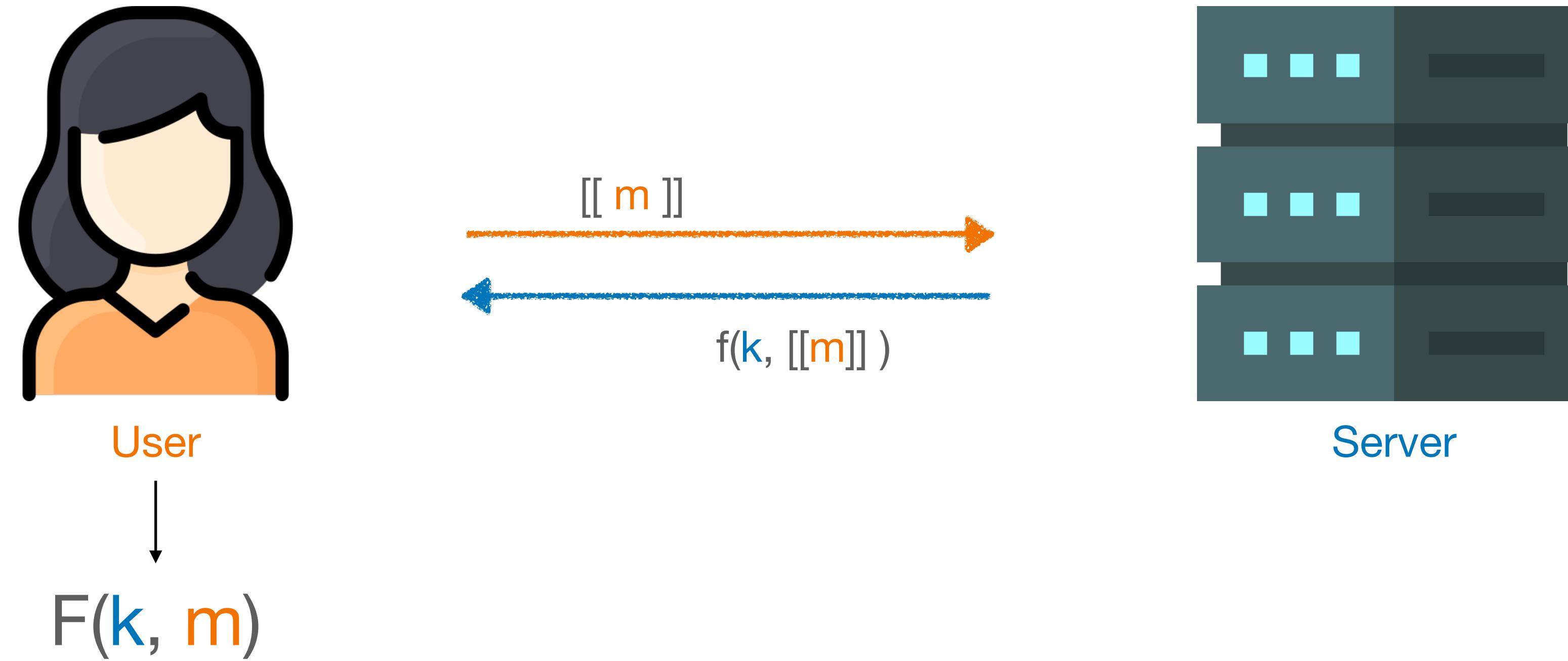


University of  
BRISTOL

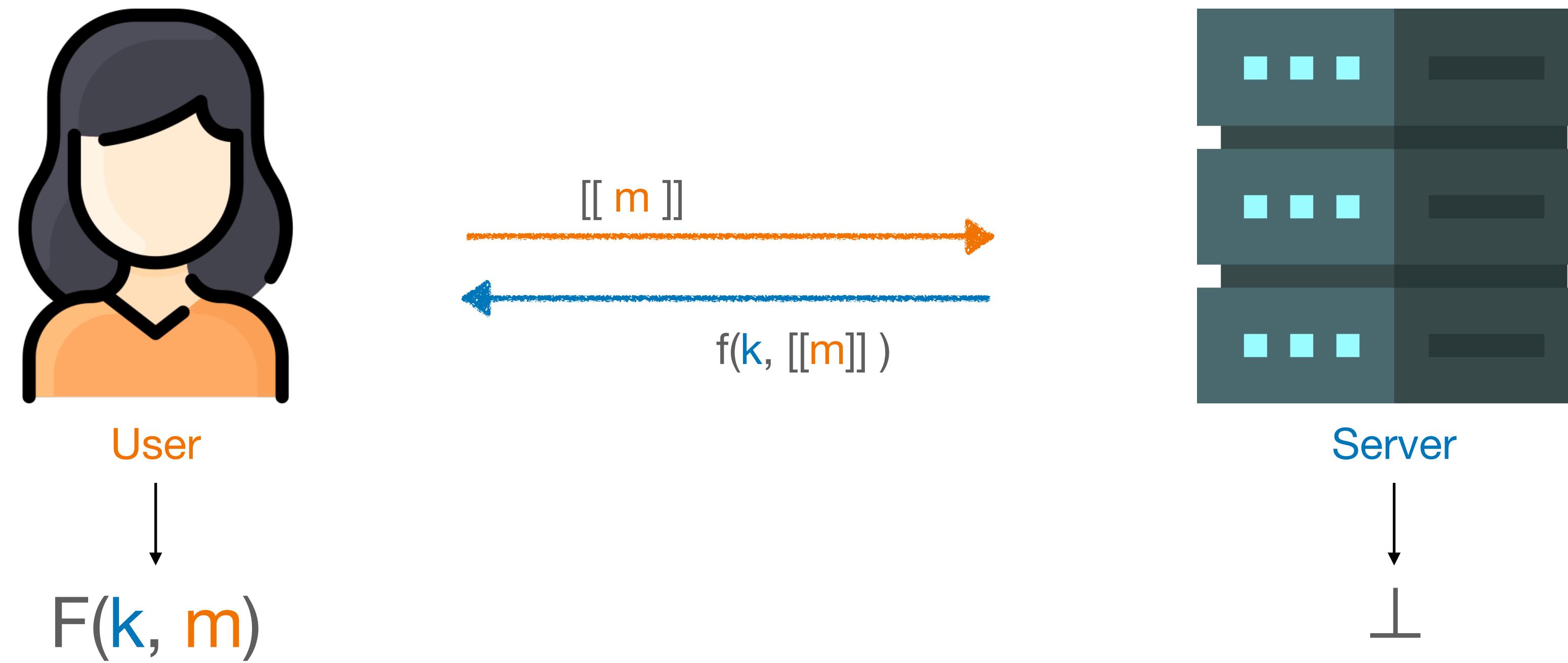
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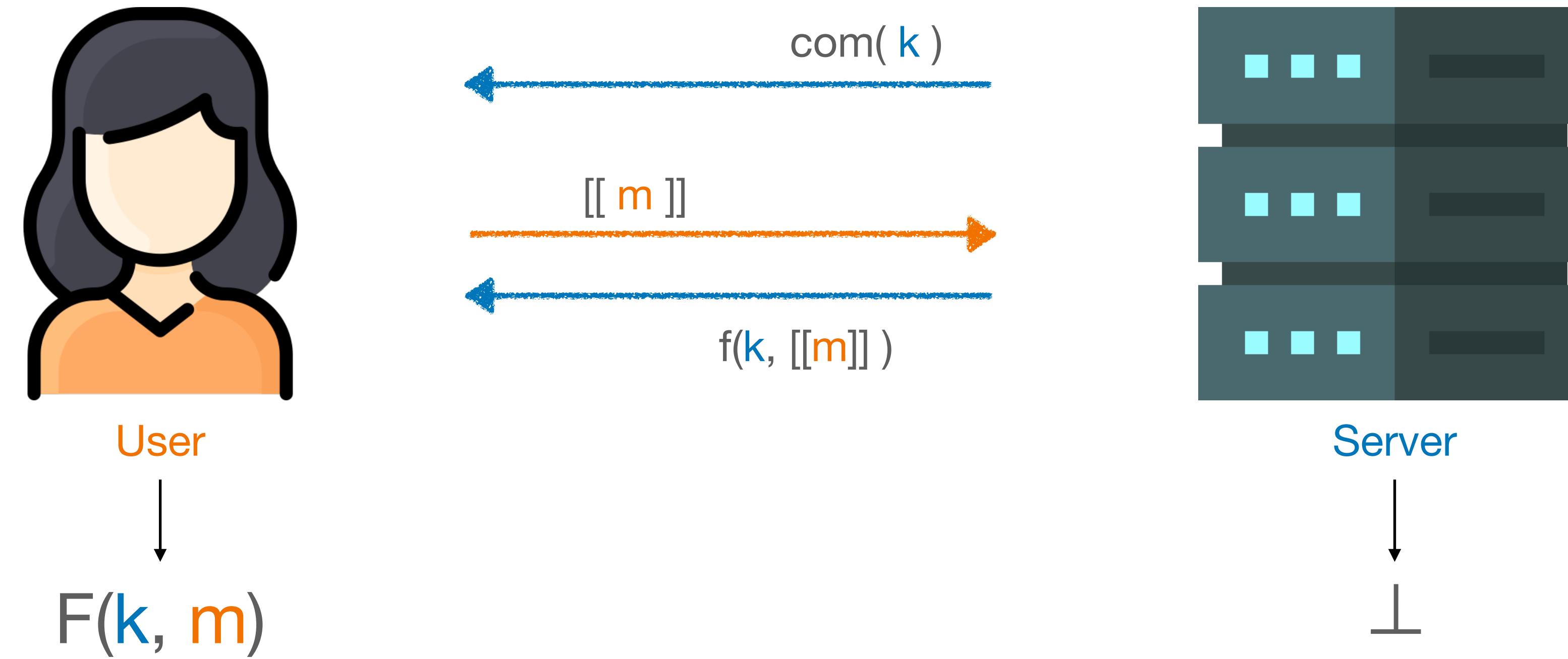
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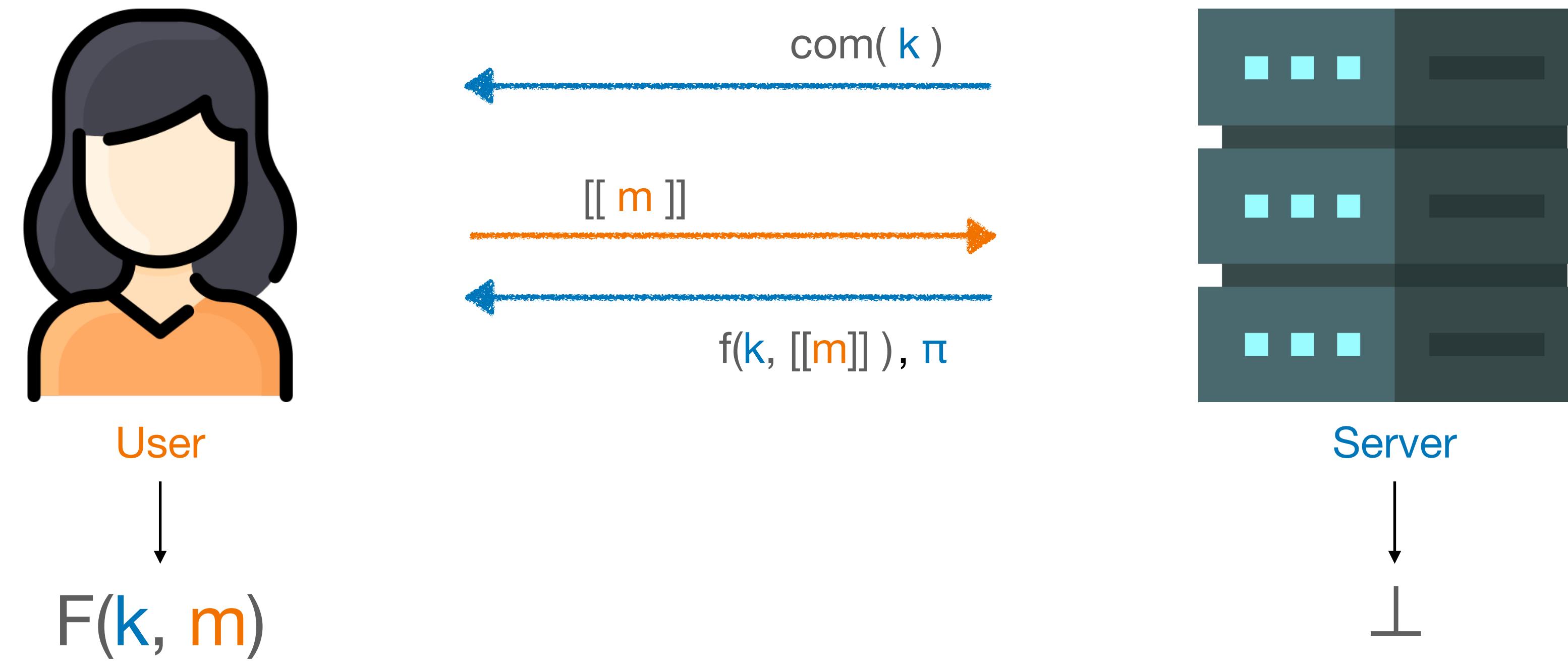
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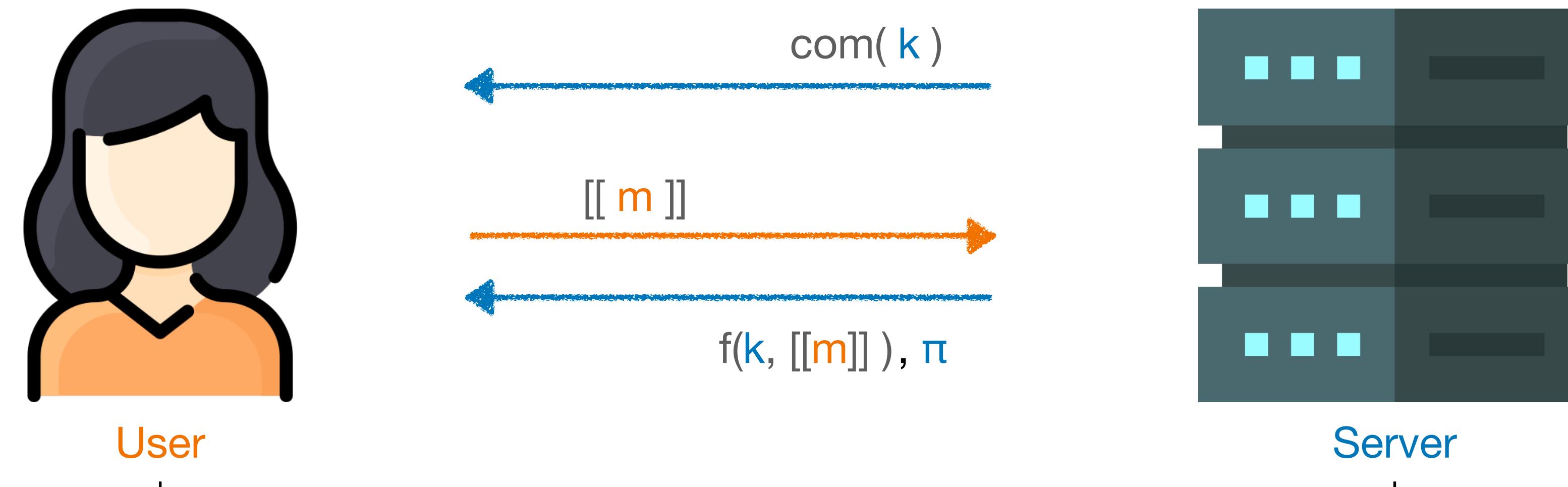
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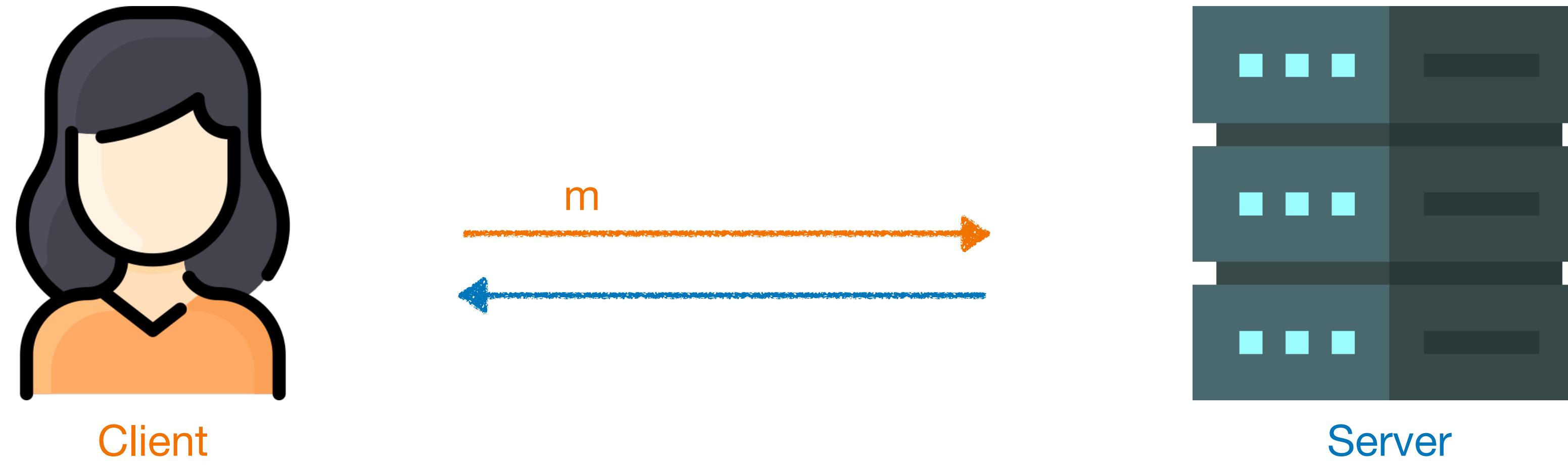
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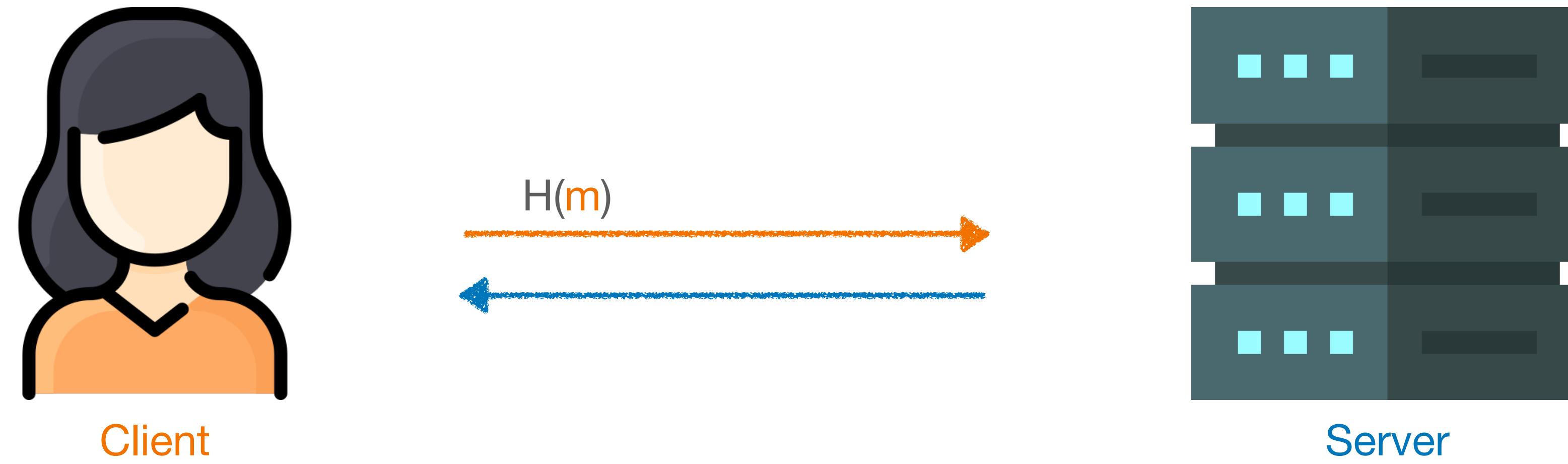
$$F(k, m)$$

- Password-checking in Microsoft Edge
- OPAQUE
- Privacy pass
- Private-set intersection
- Adaptive OT
- ....

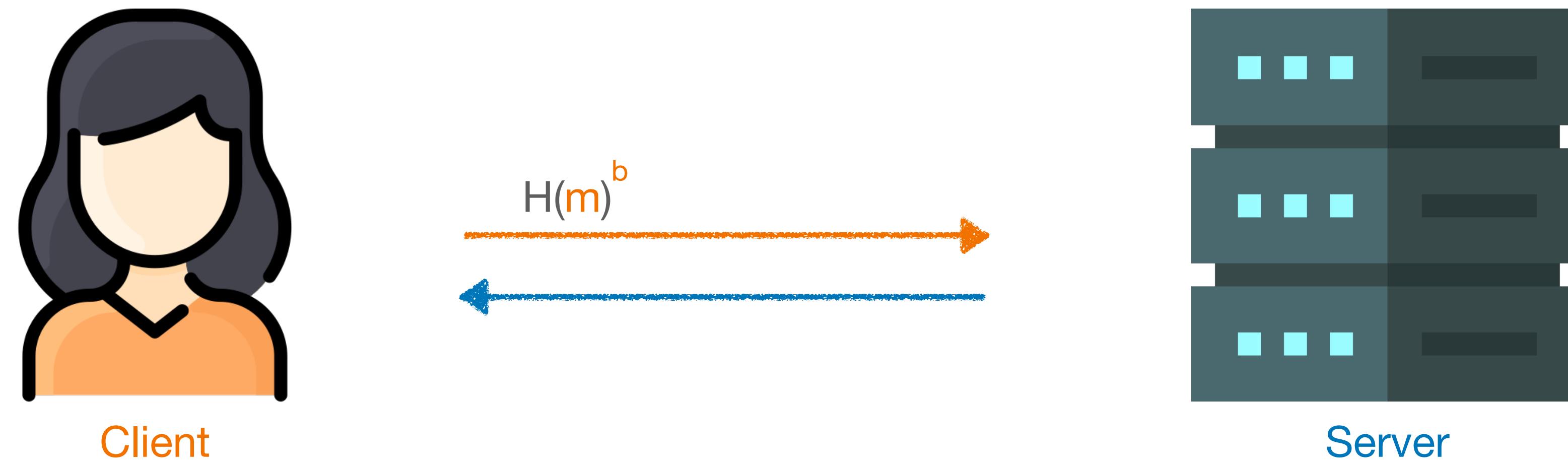
# HashDH OPRF



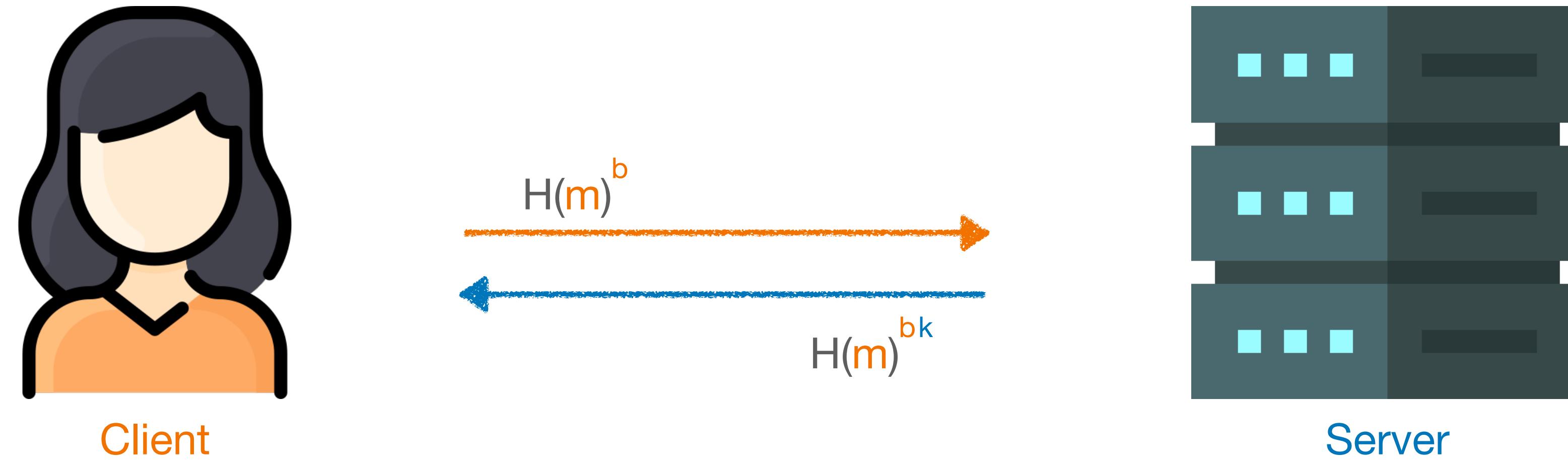
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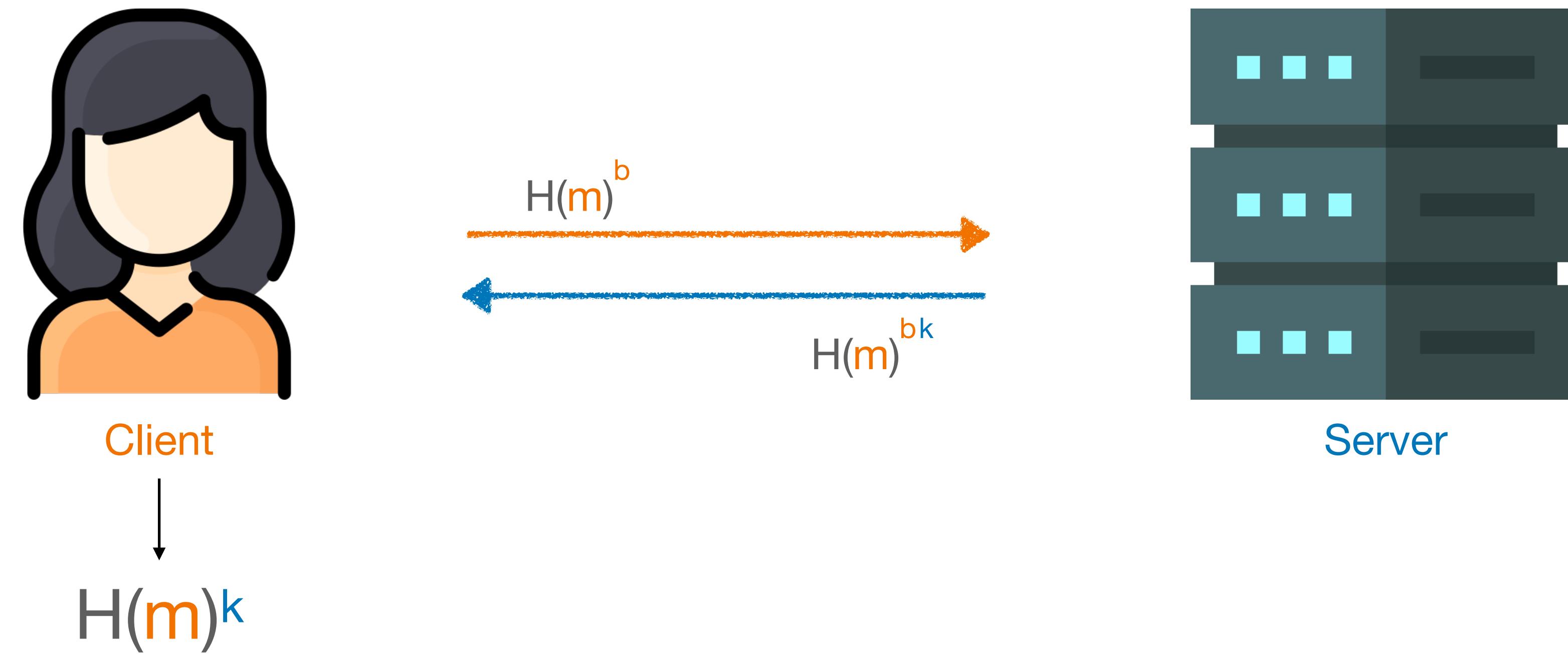
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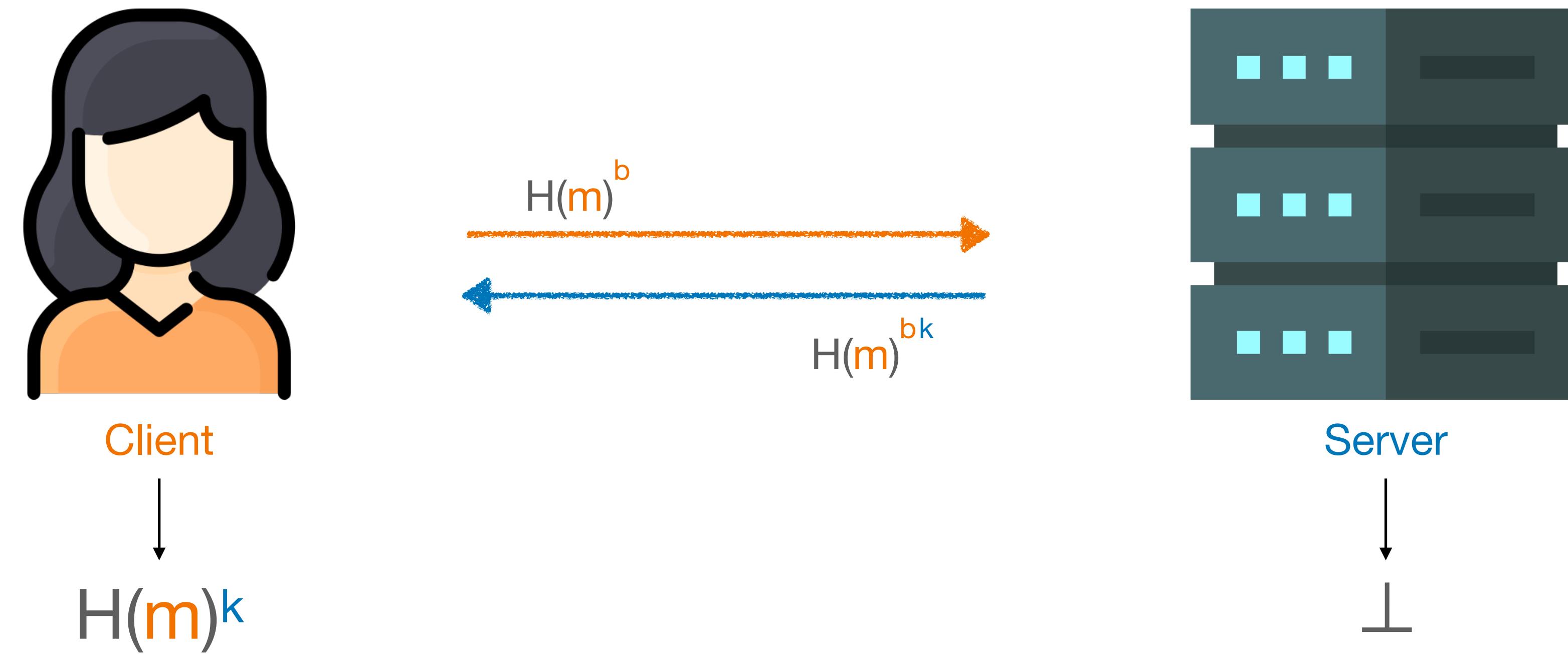
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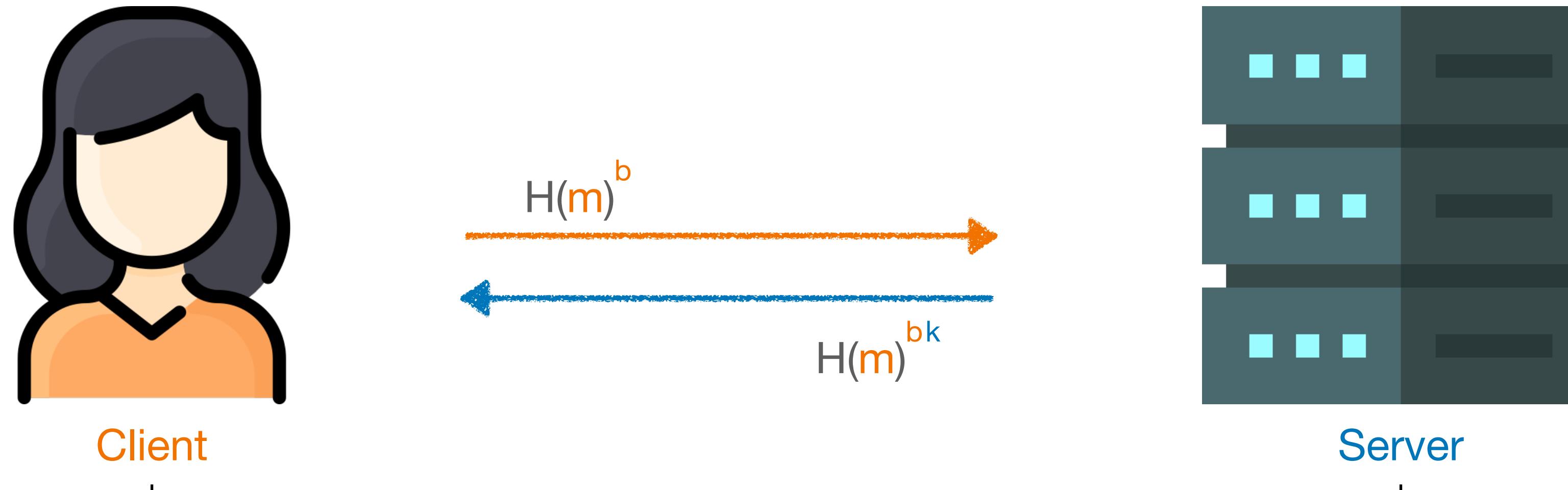
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- Server doesn't learn anything ✓
- Output is deterministic ✓
- Client only learns one output ✓

# Post-quantum OPRFs

- Generic MPC techniques



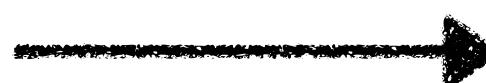
- many rounds (can't be optimal)

- VOPRF based on lattices [ADDS19]



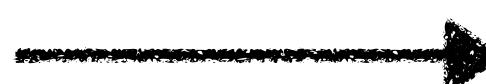
- round optimal
- feasibility result ( $> 2^{40}$  bits of comms)

- VOPRF based on SIDH [BKW20]



- six rounds
- broken by attack on PR and on SIDH

- OPRF based on CSIDH [BKW20]



- three rounds (OT required)
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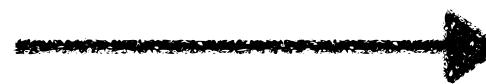
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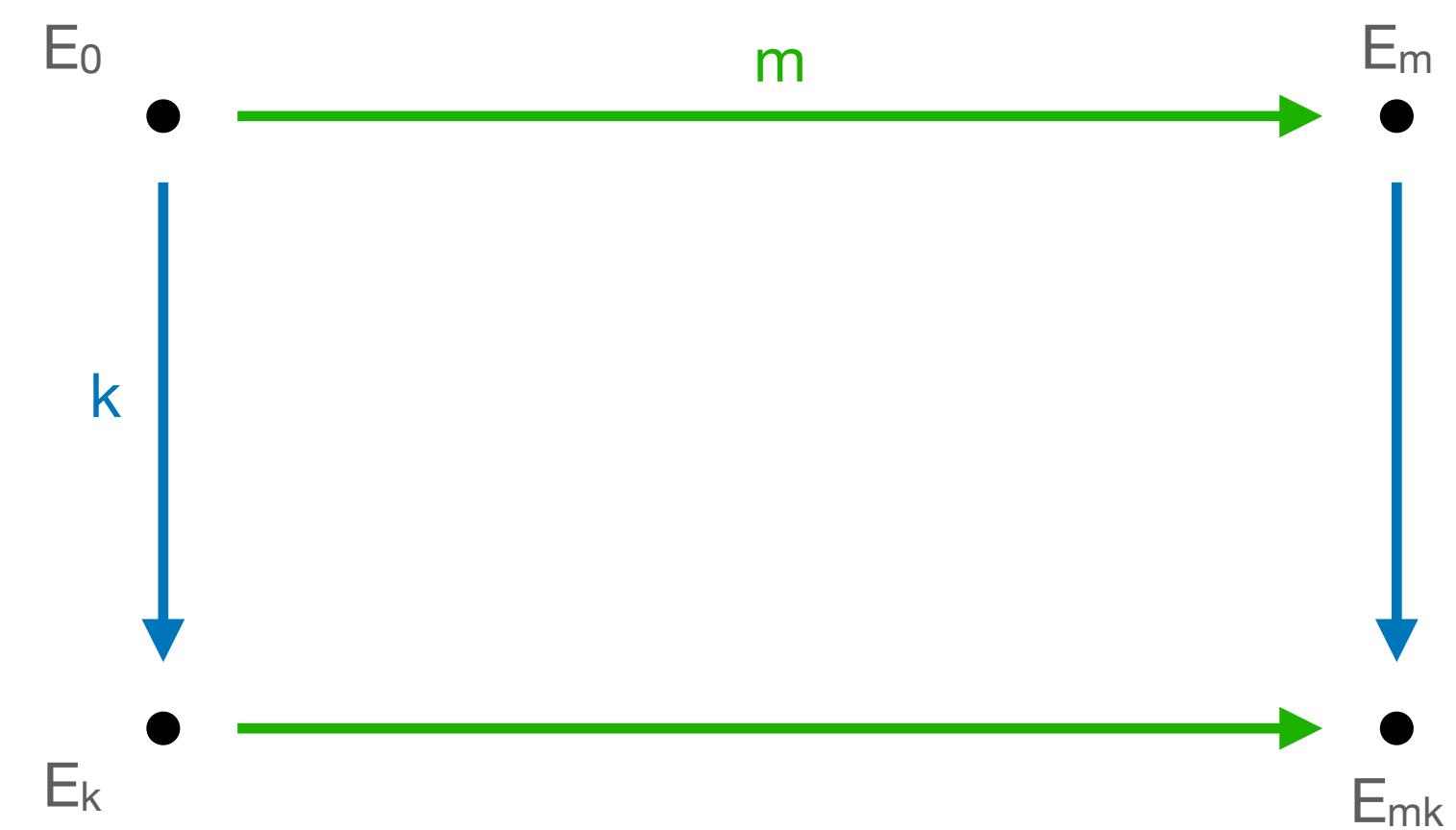
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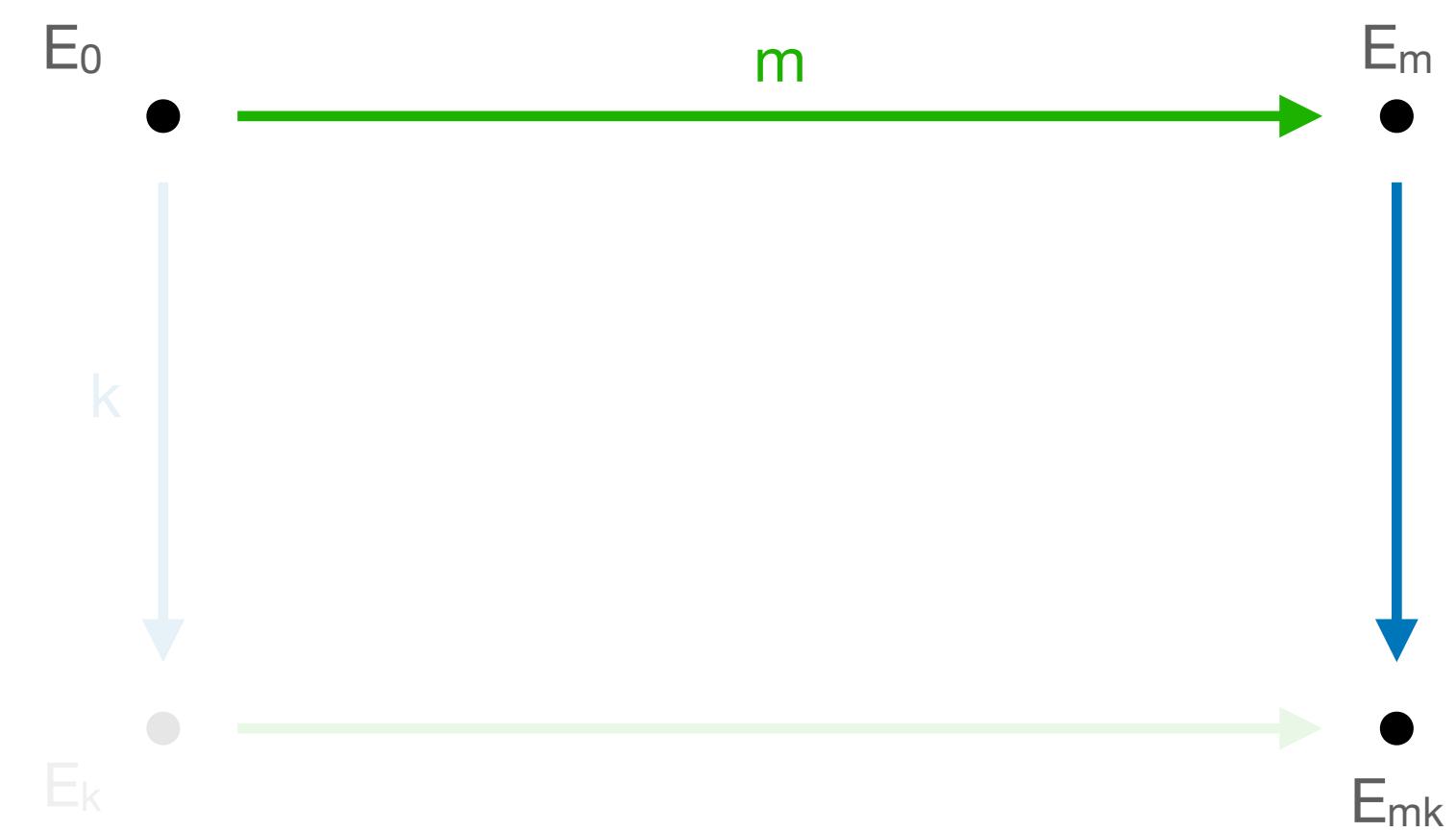


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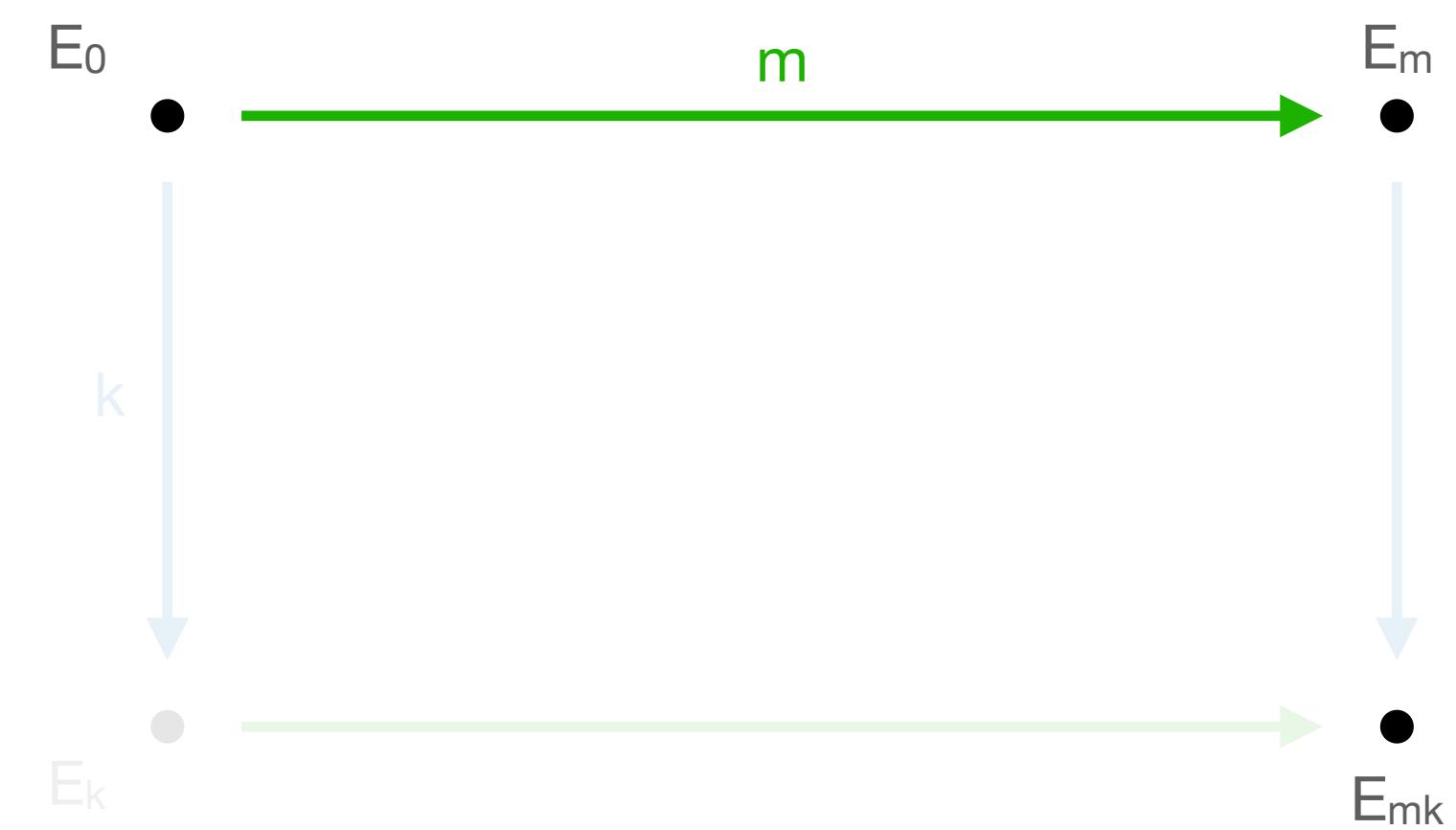
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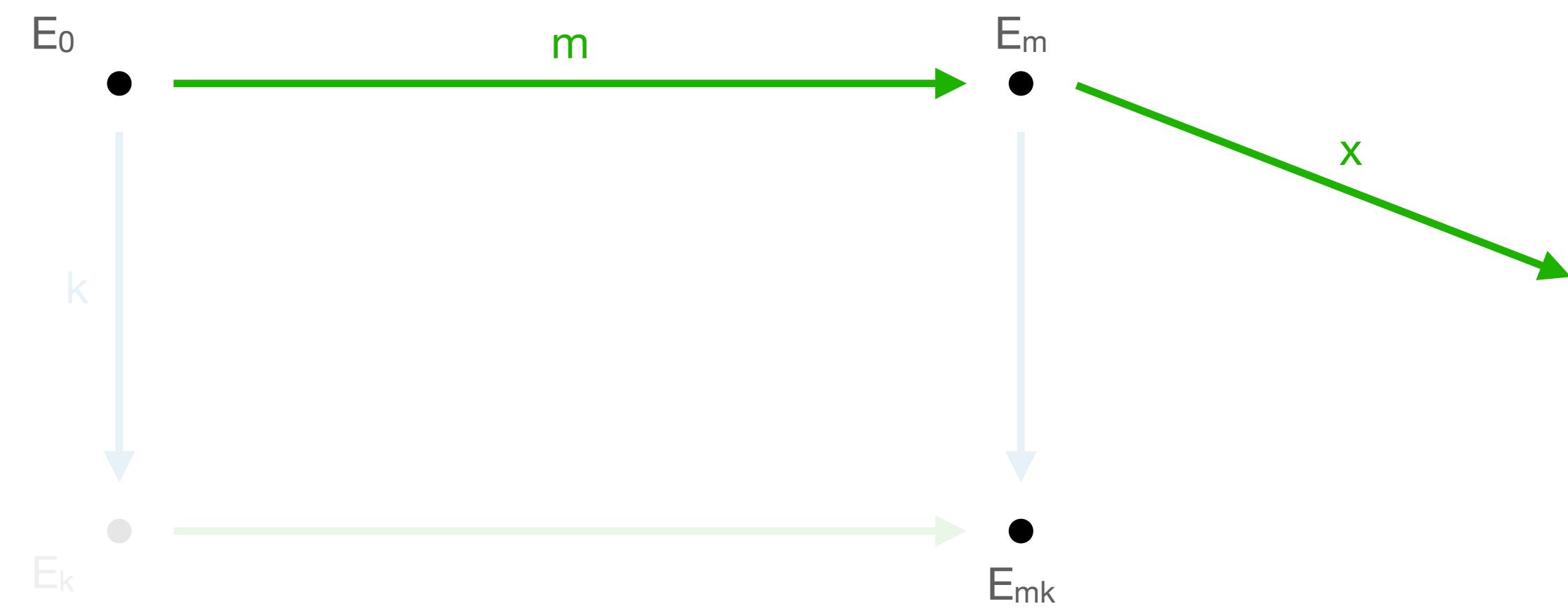
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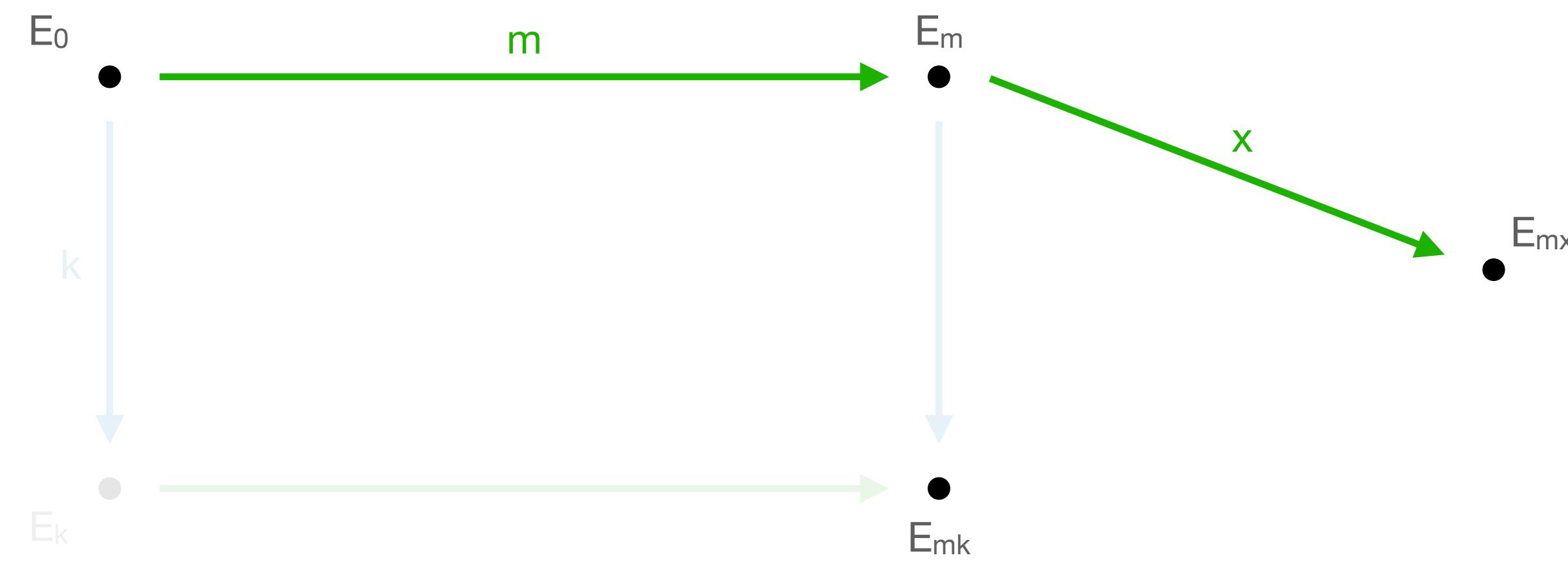
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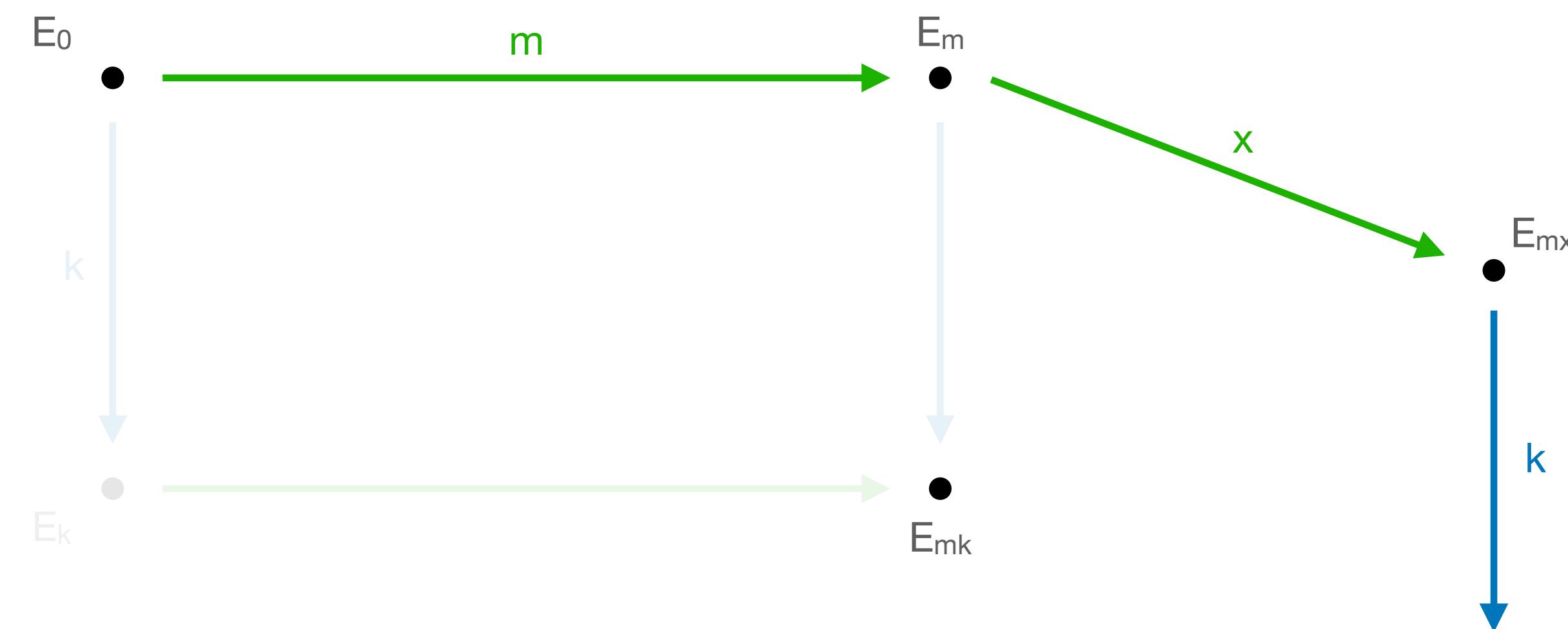
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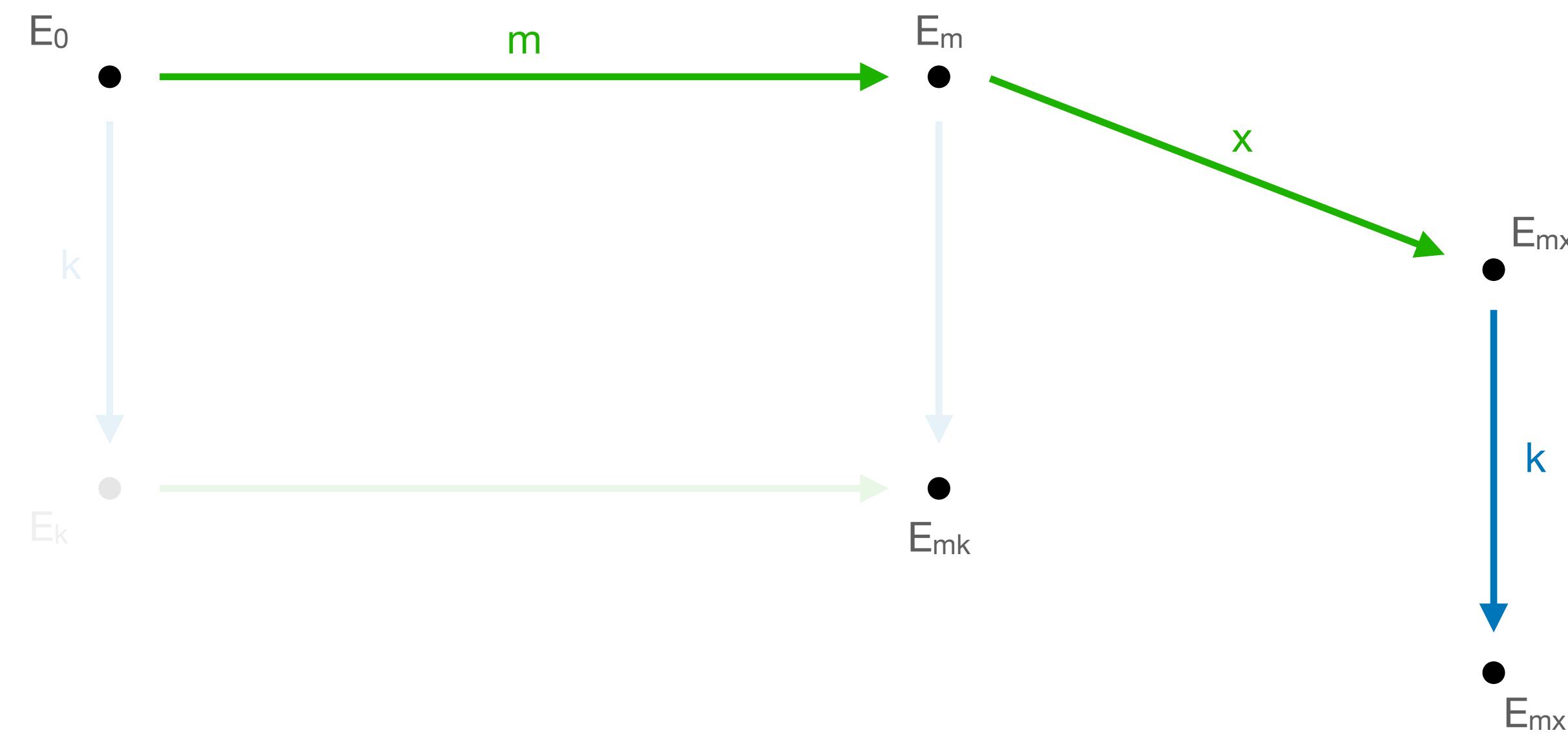
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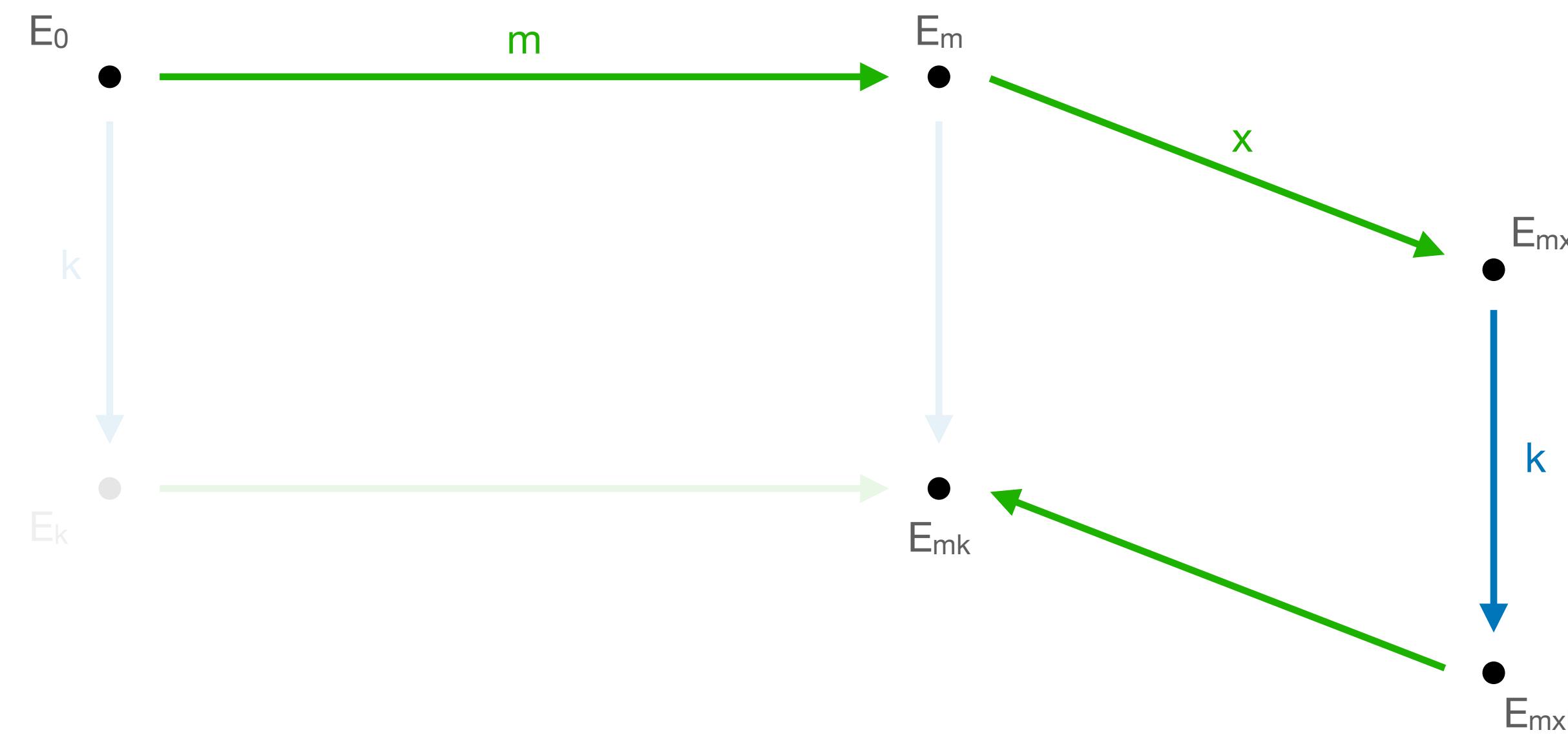
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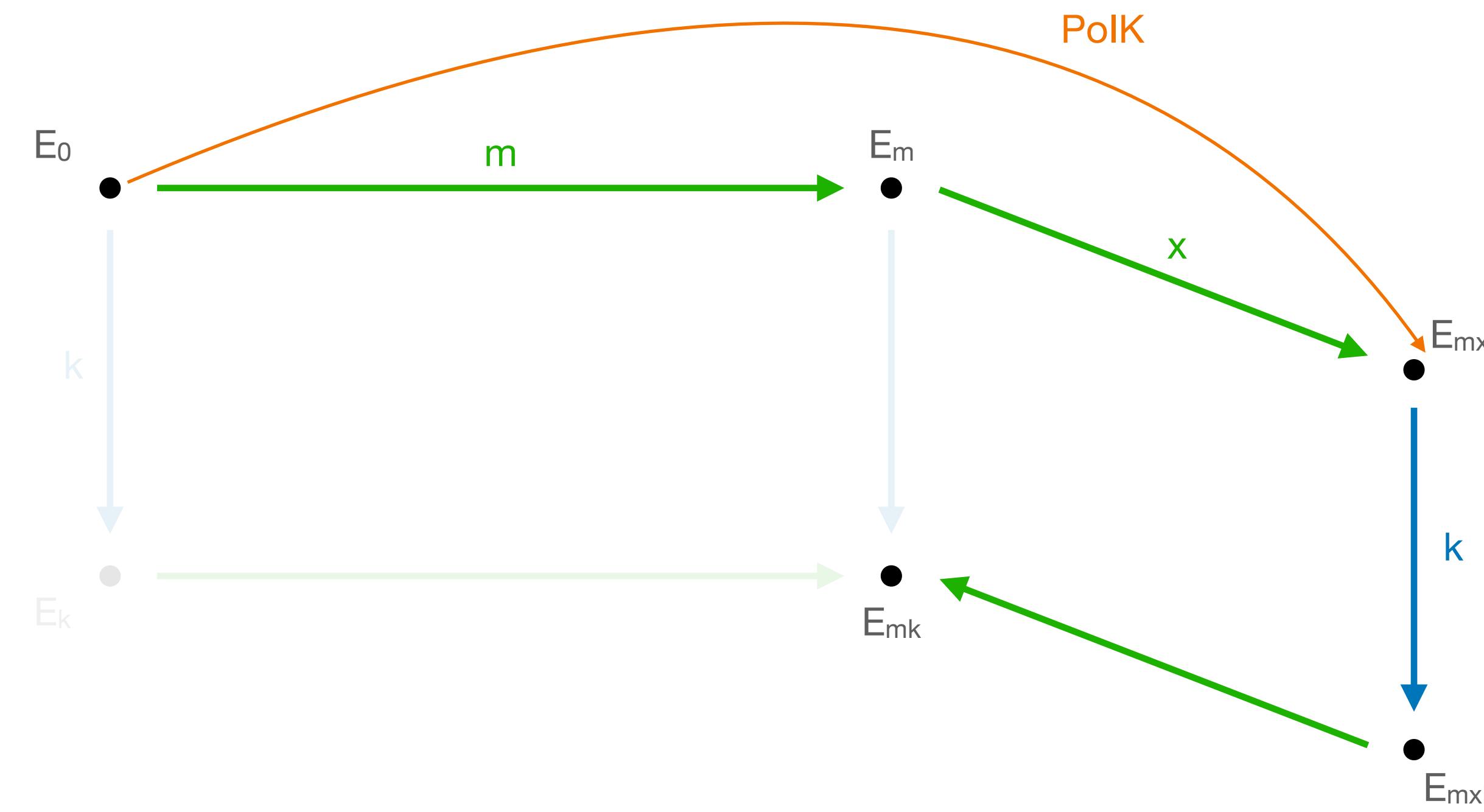
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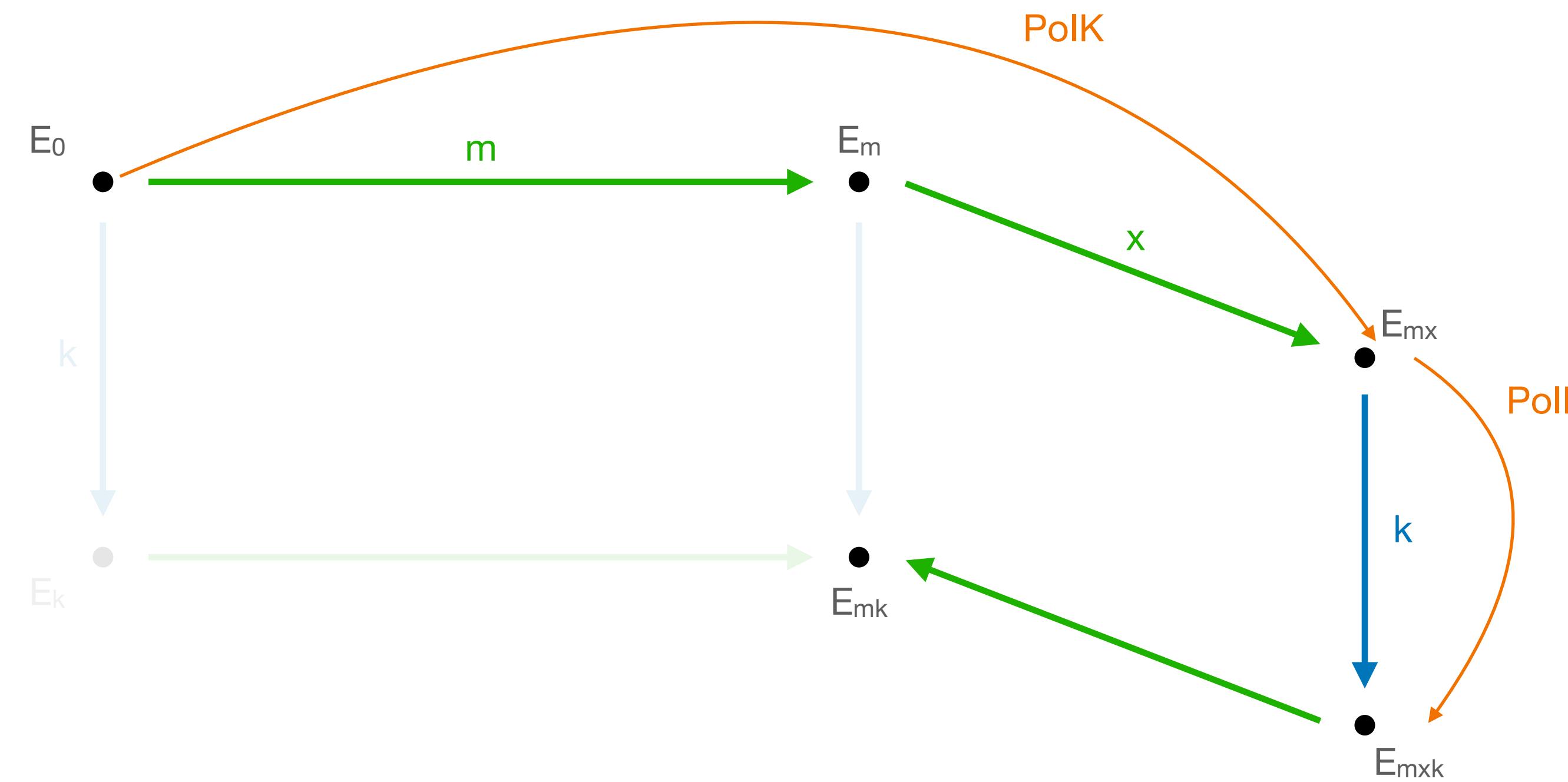
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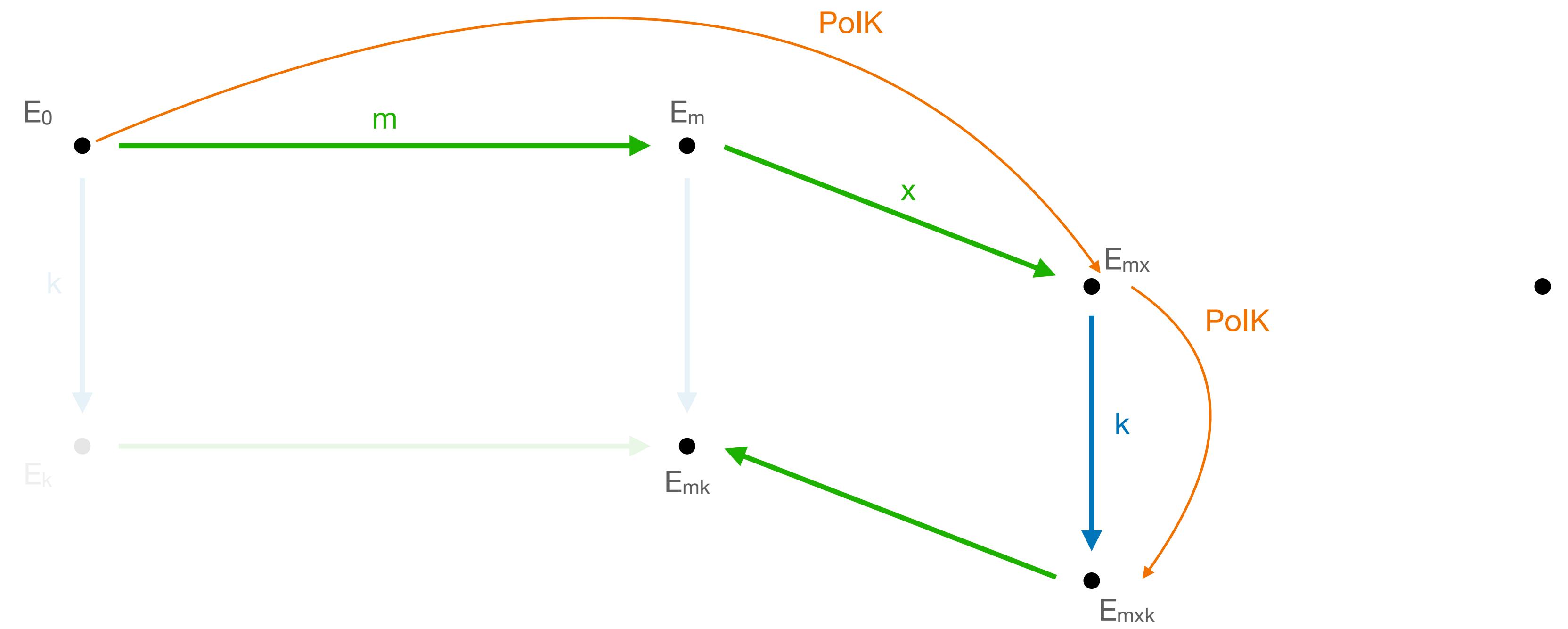
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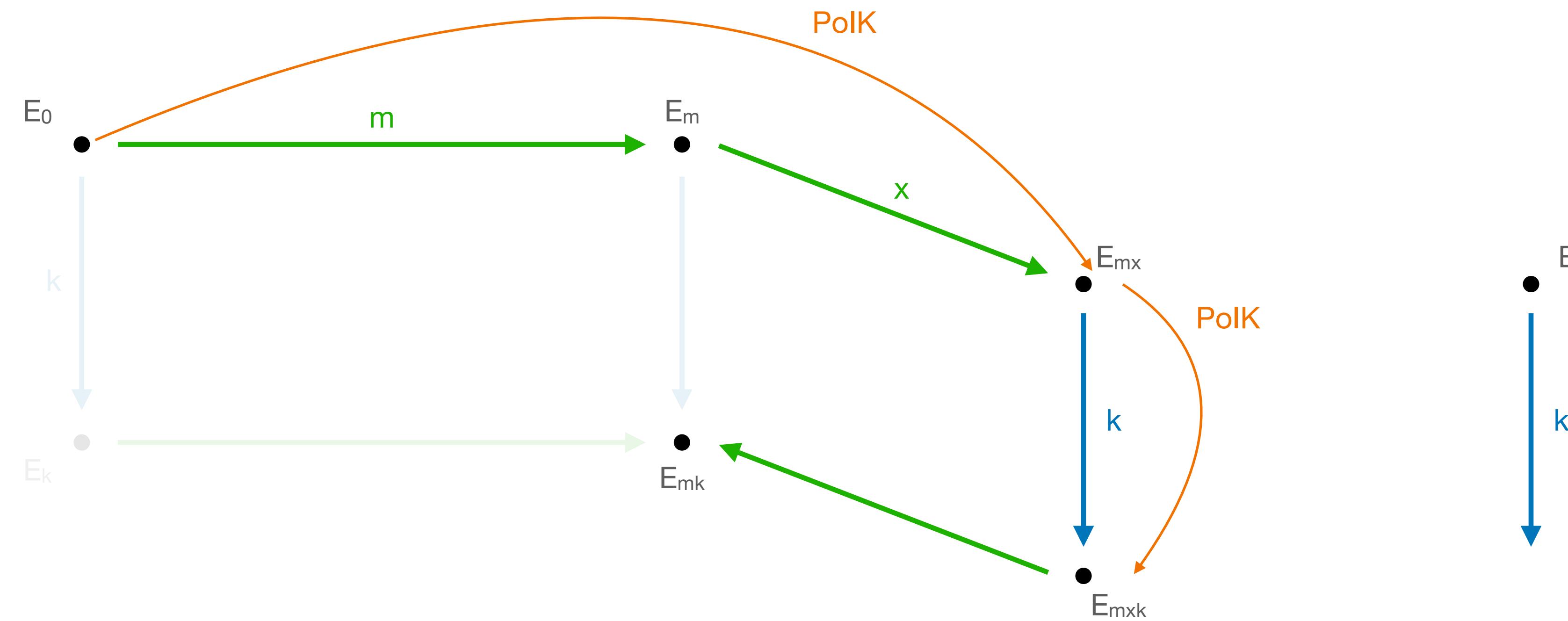
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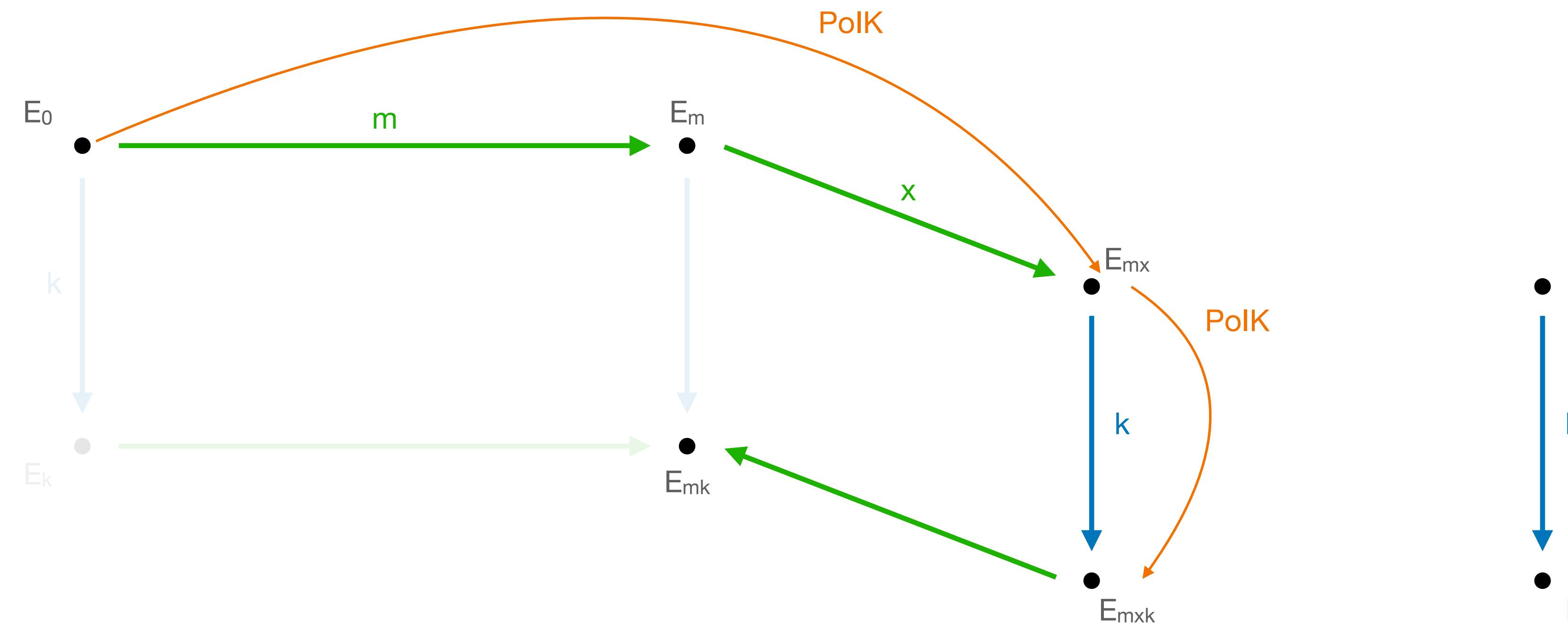
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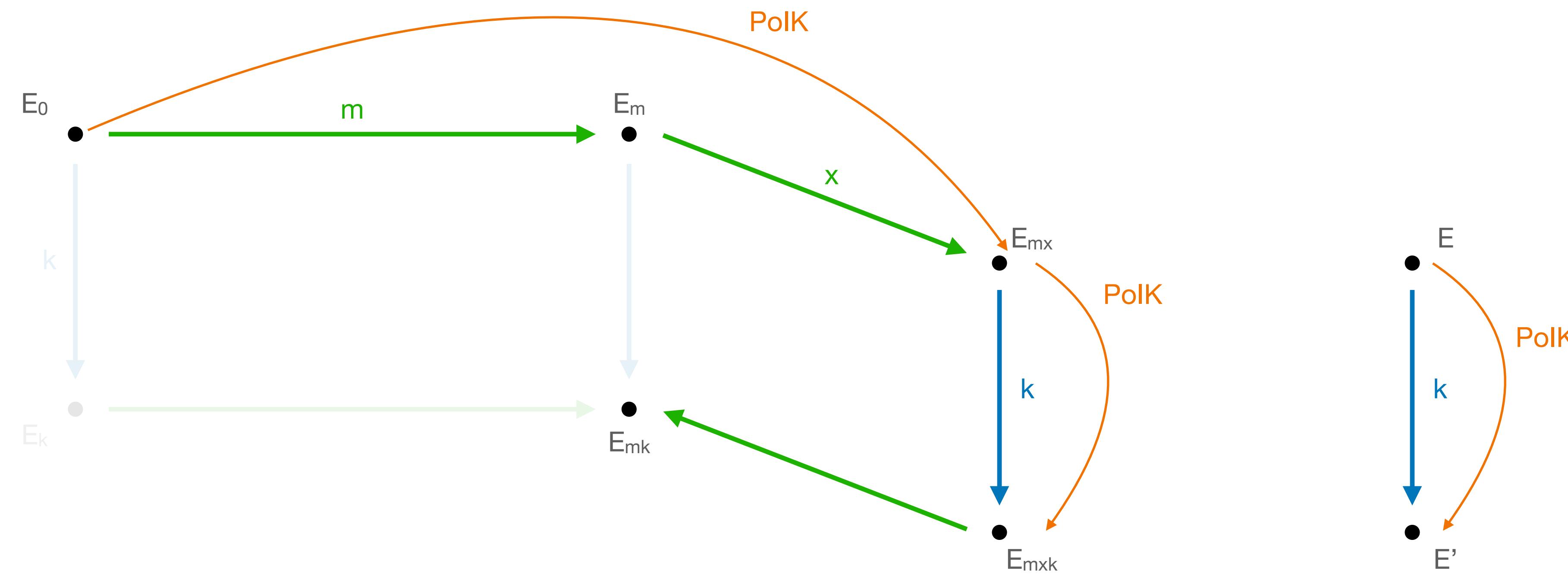
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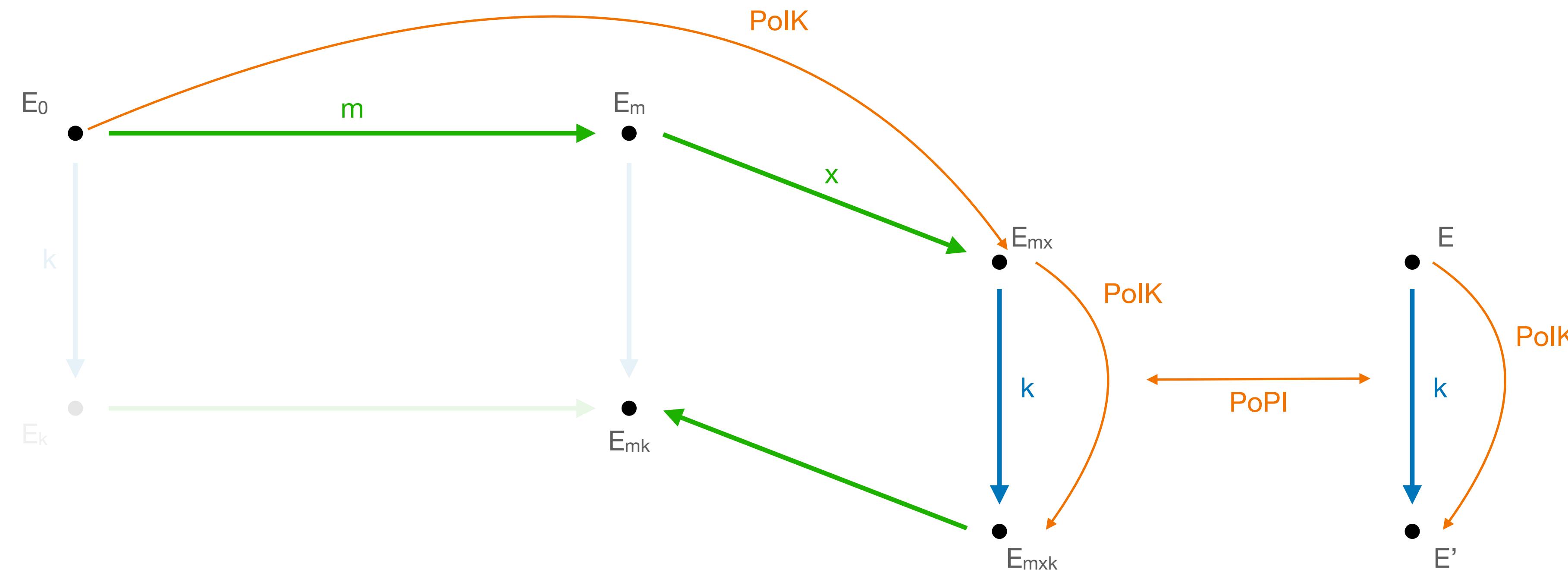
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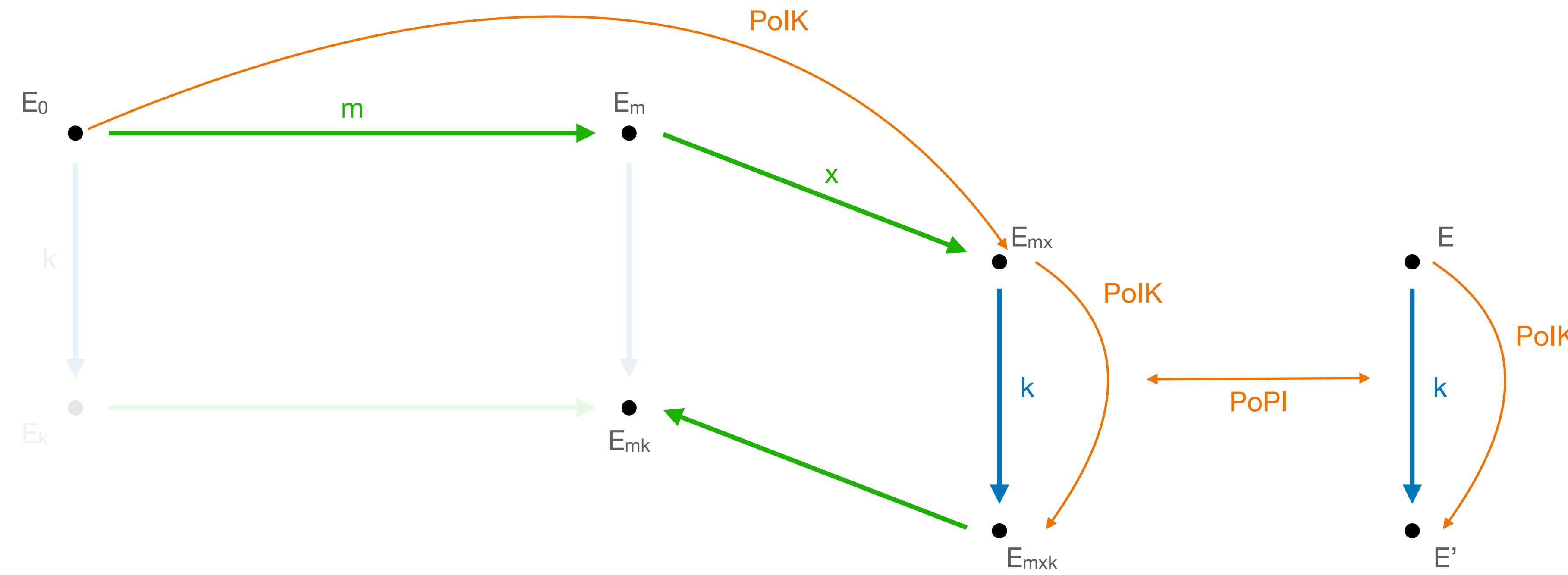
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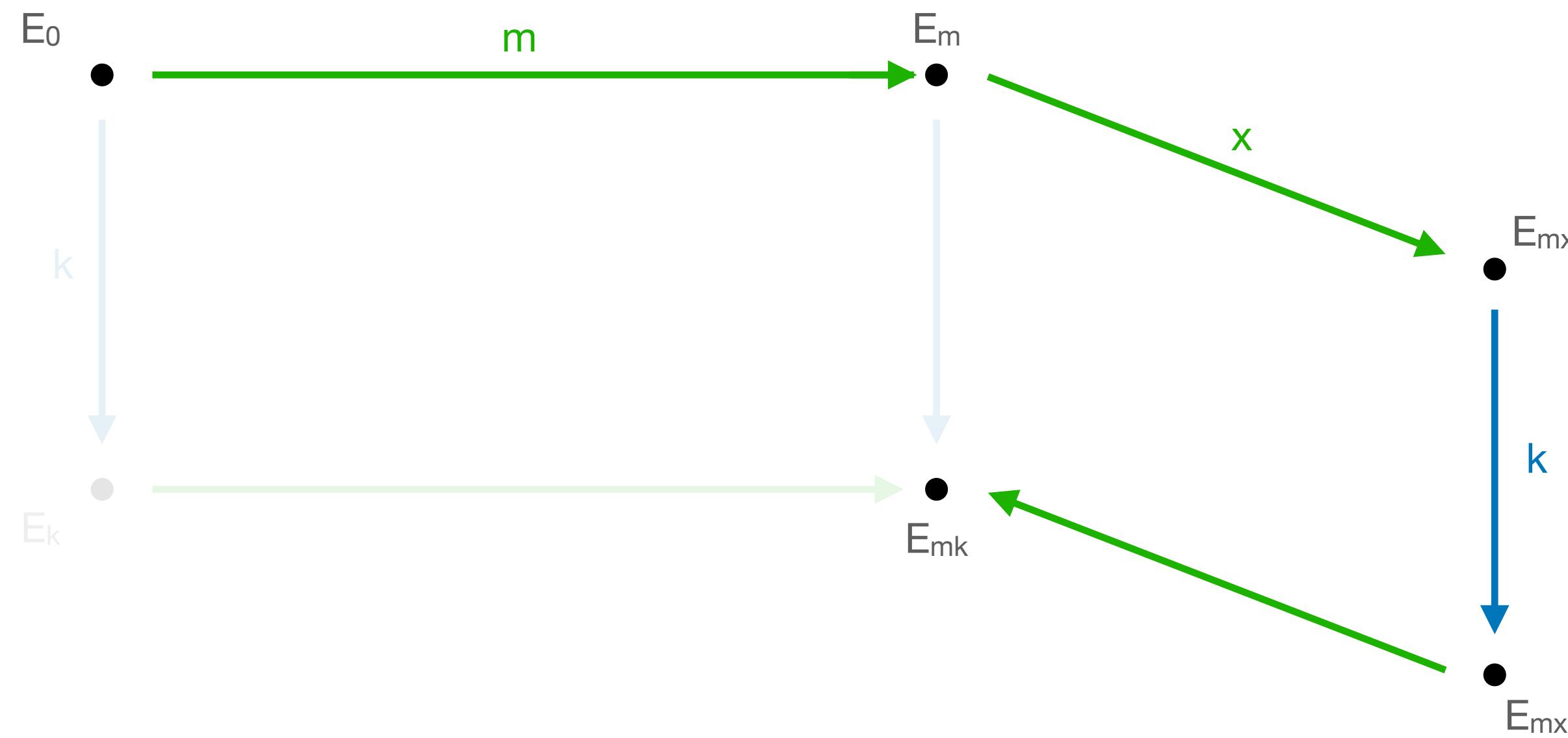
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$$F(k, m) = H(m, j_{mk}, E')$$

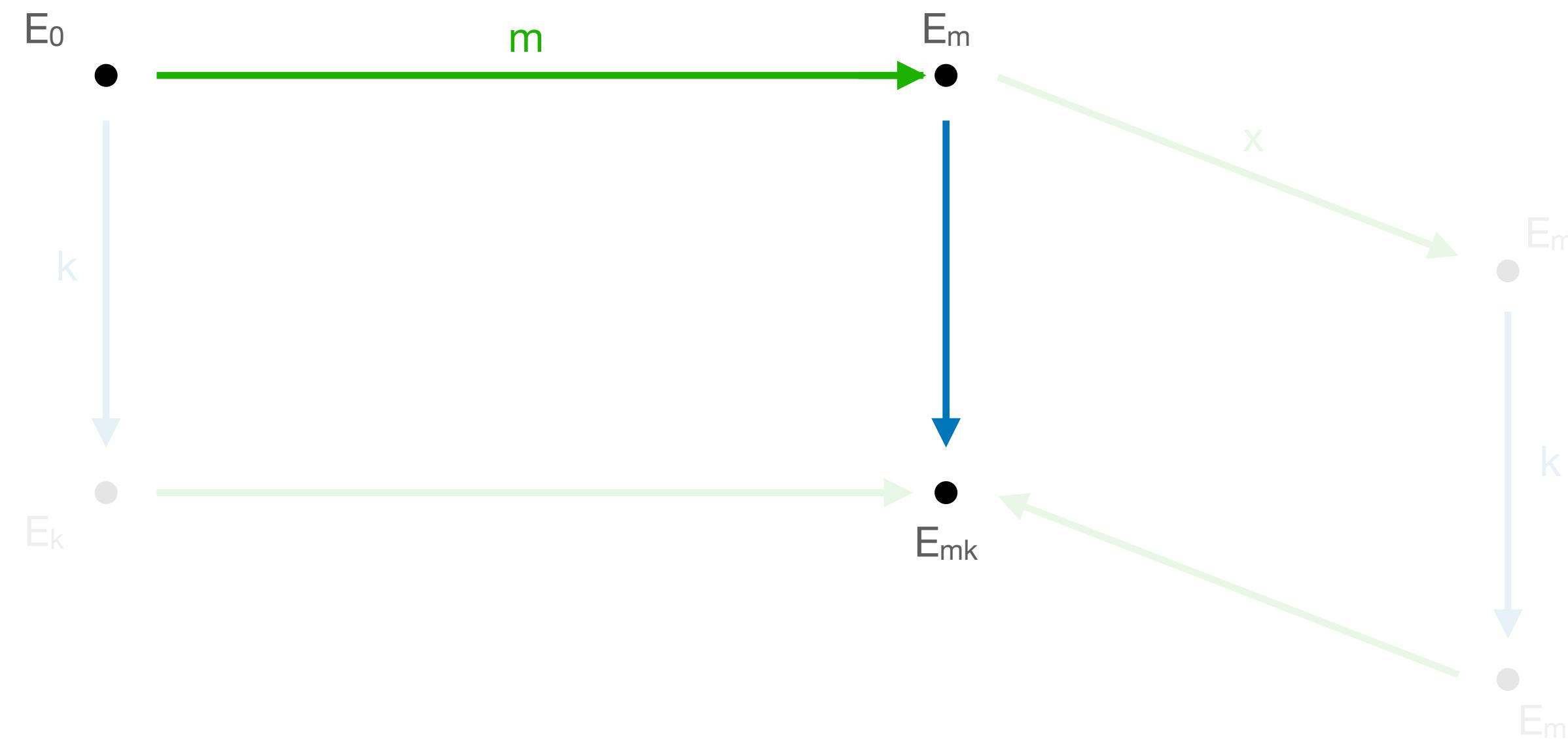
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Pseudorandomness: after  $n$  interactions, an attacker cannot generate  $n+1$  PRF outputs



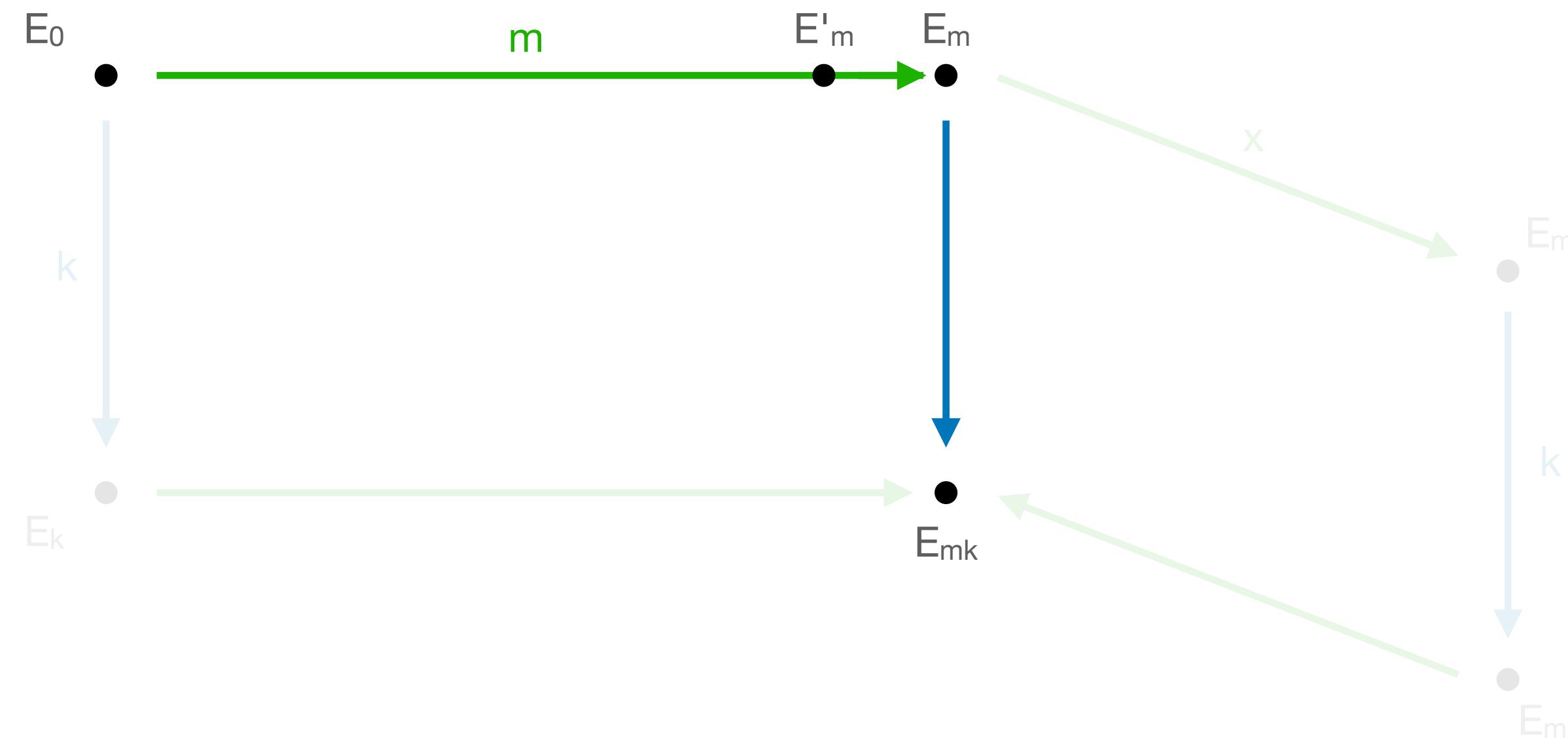
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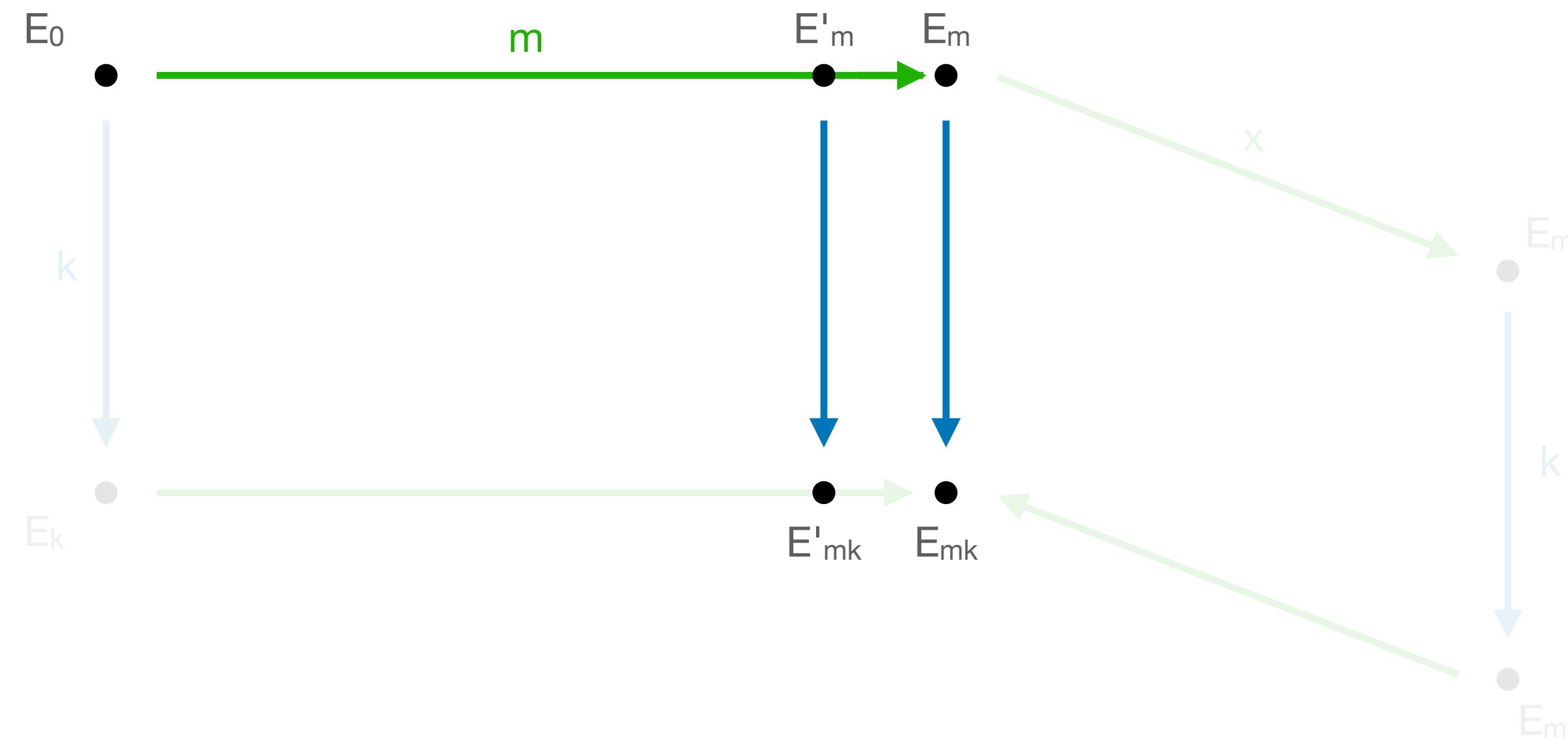
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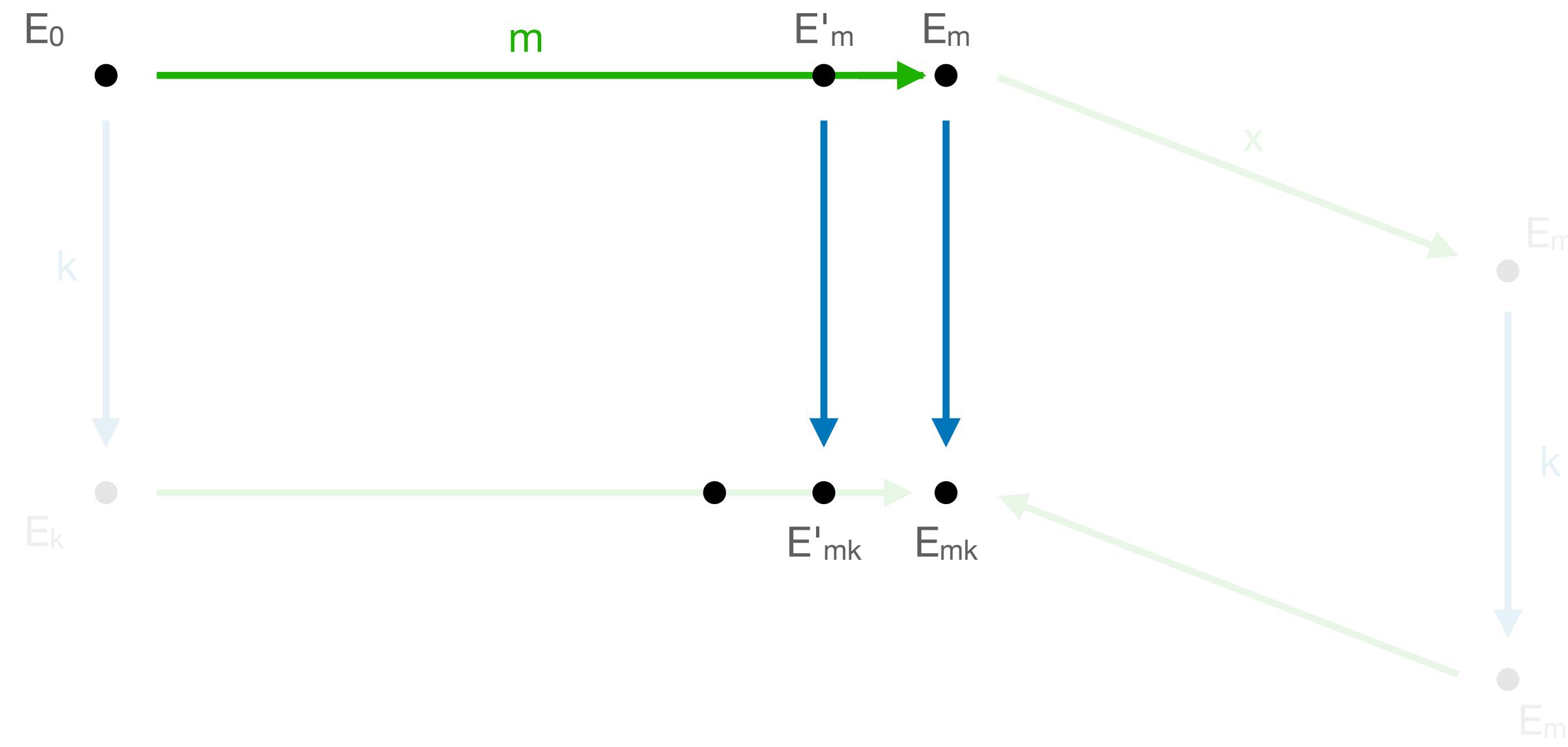
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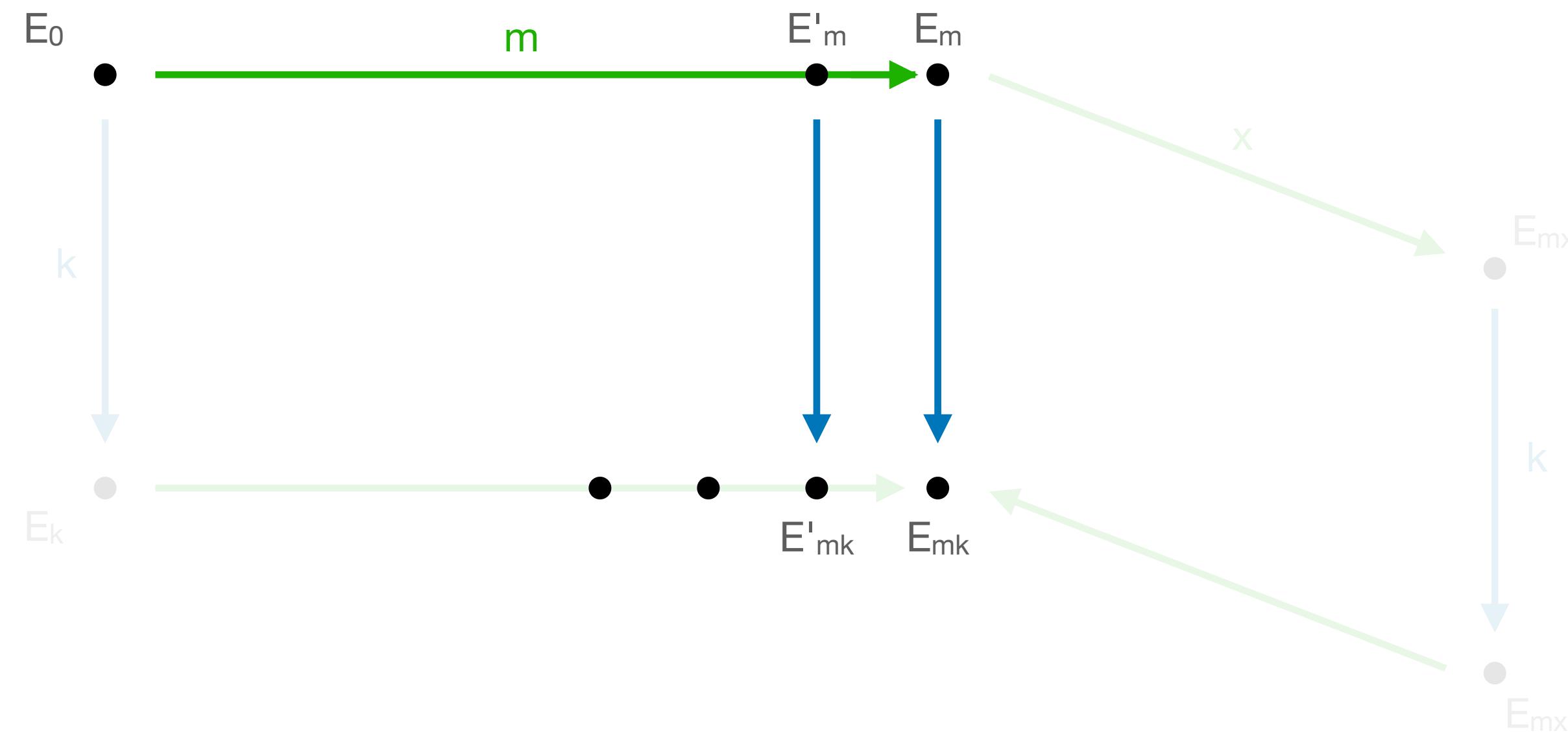
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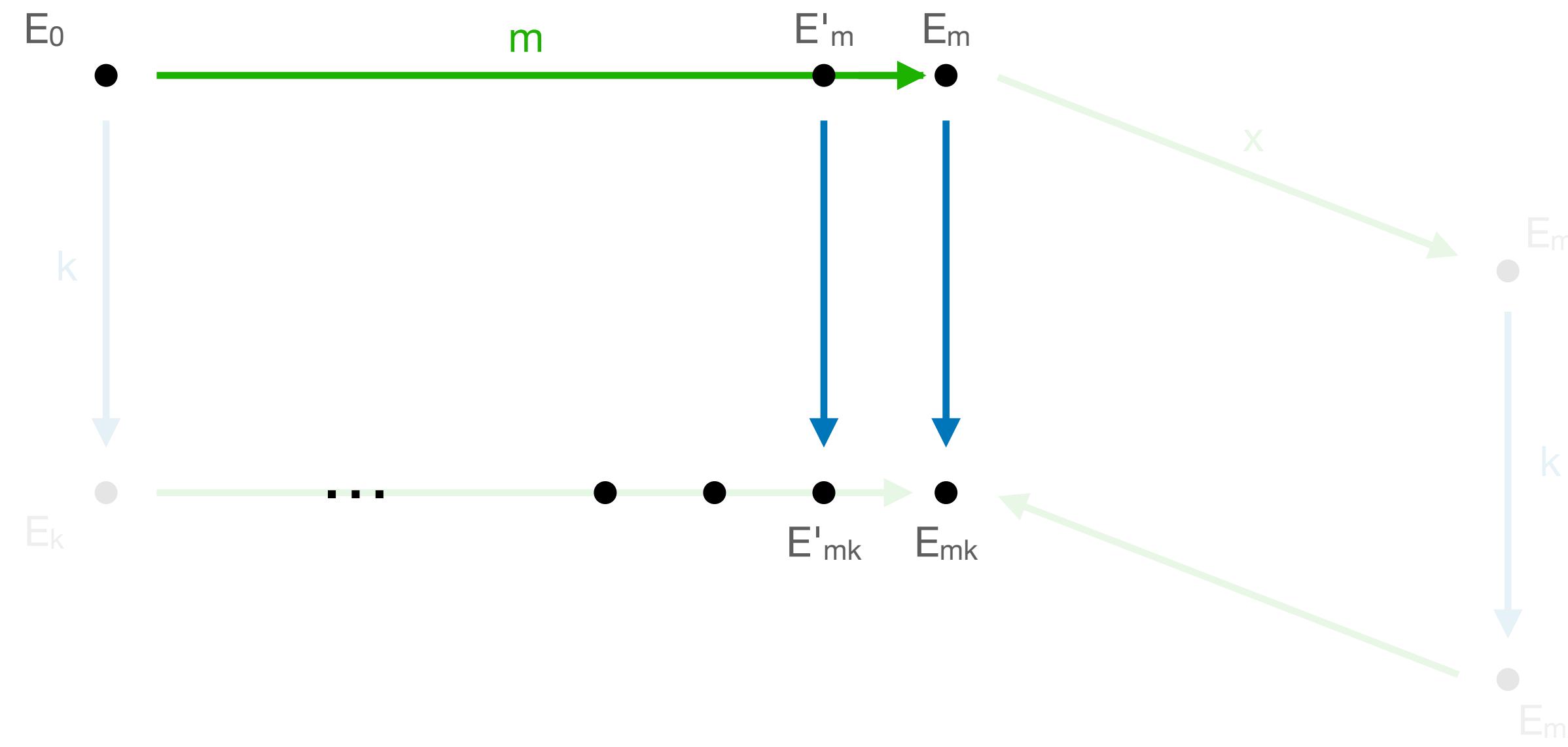
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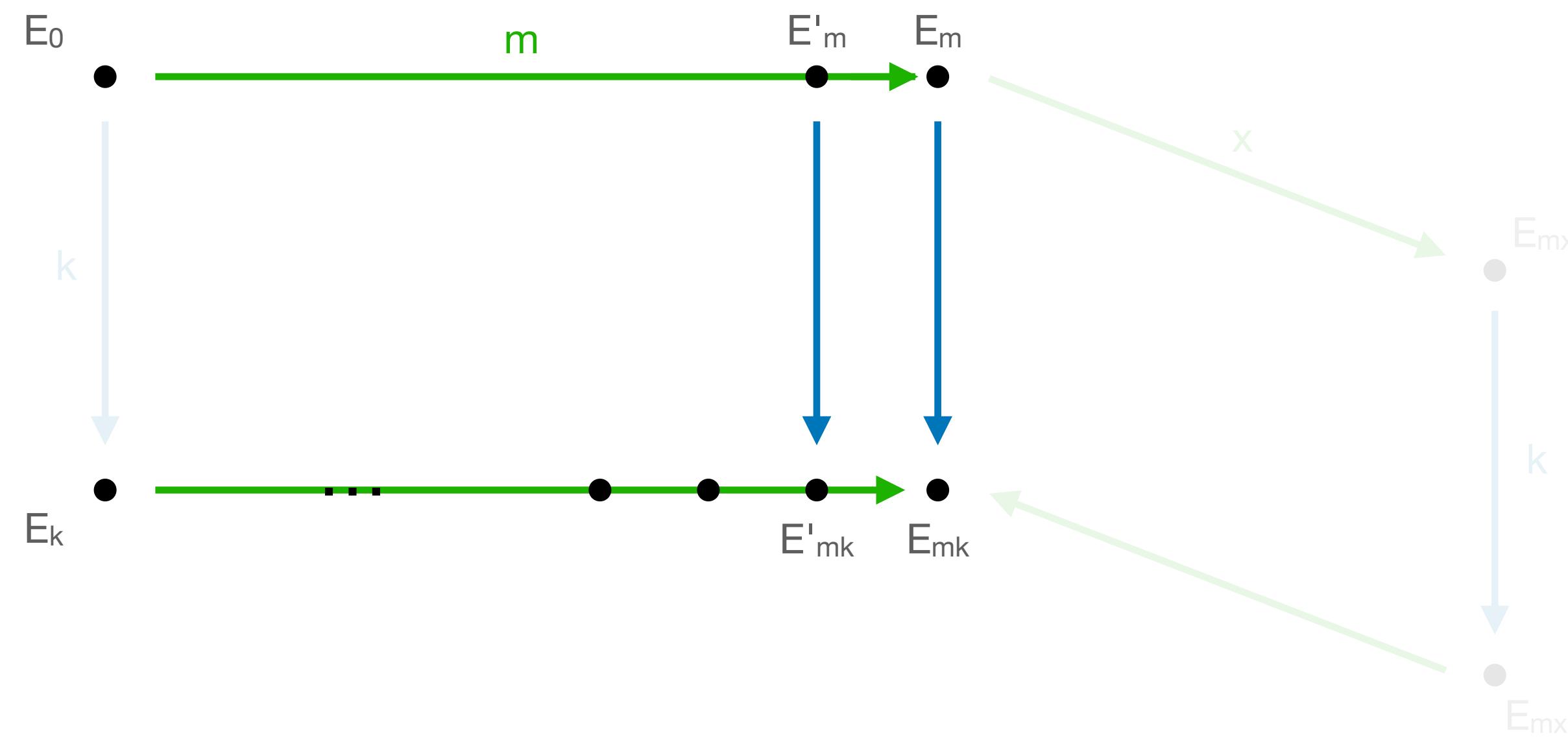
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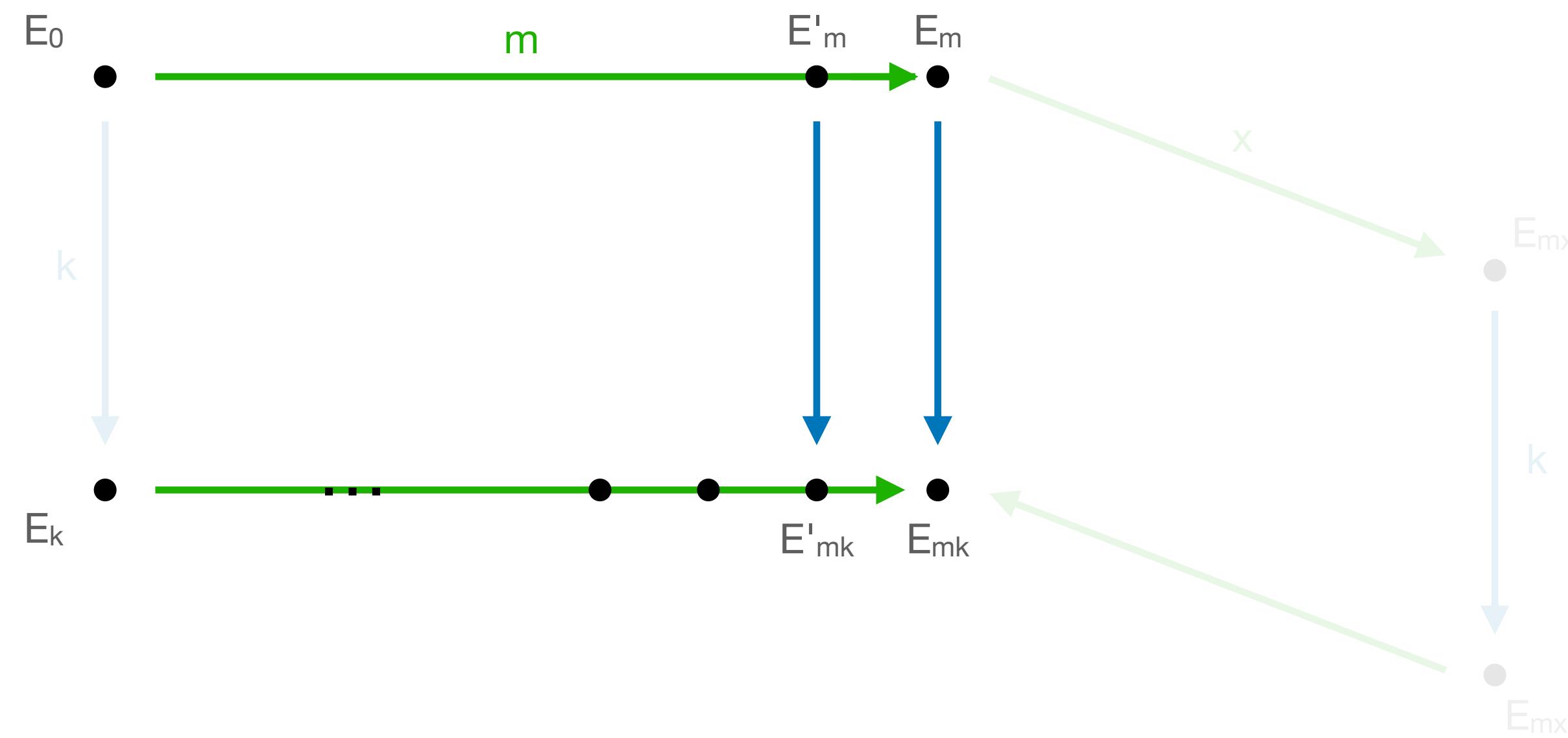
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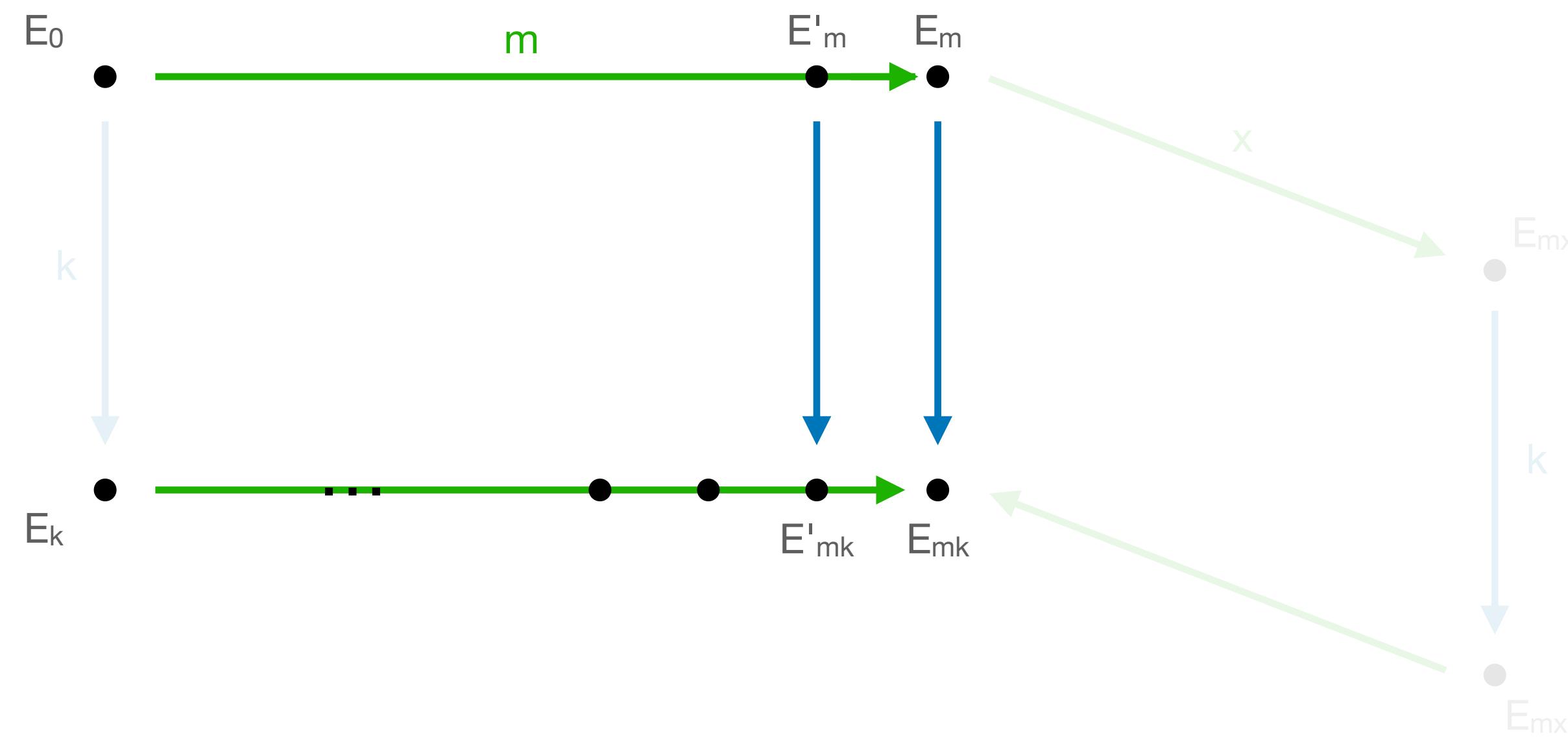
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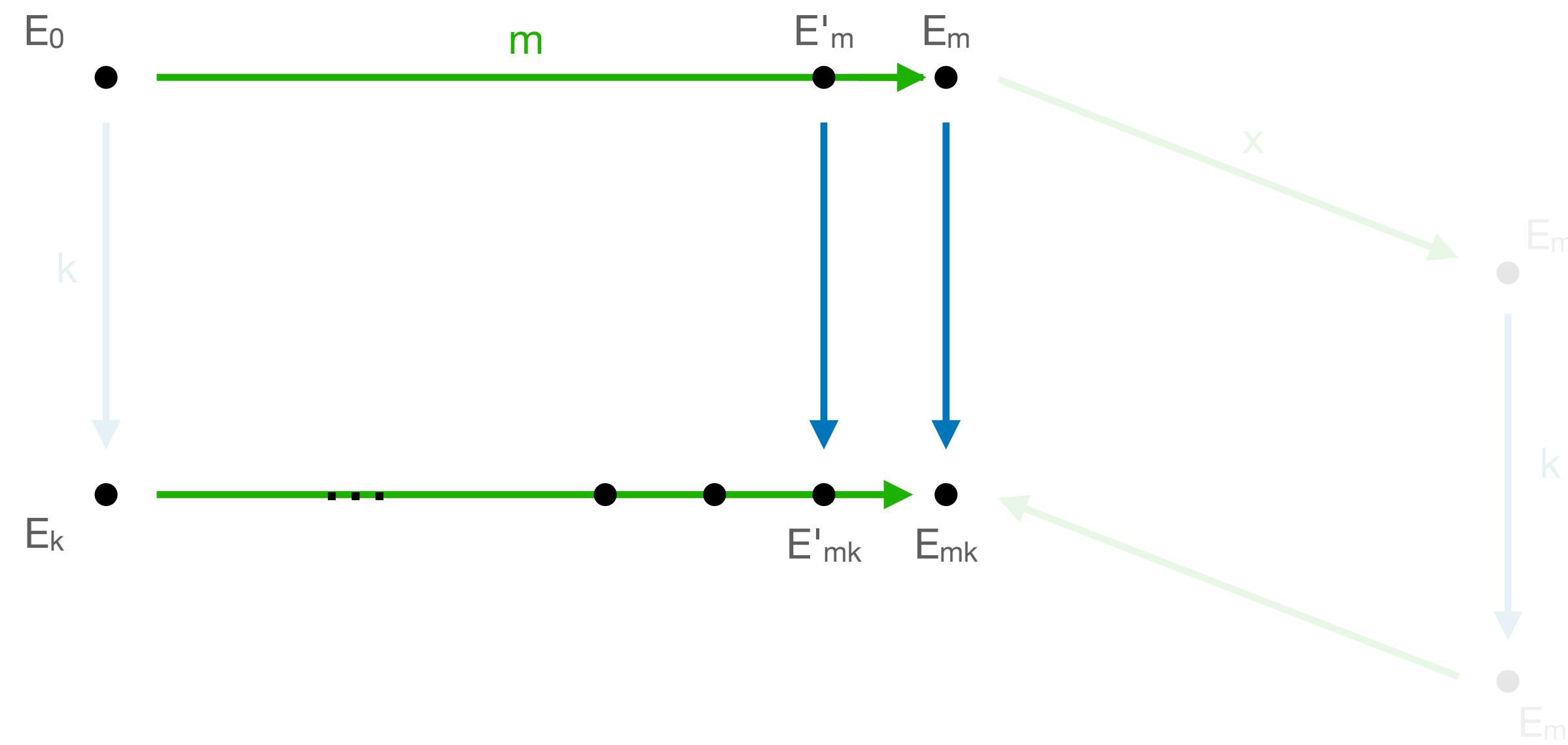


Part 2

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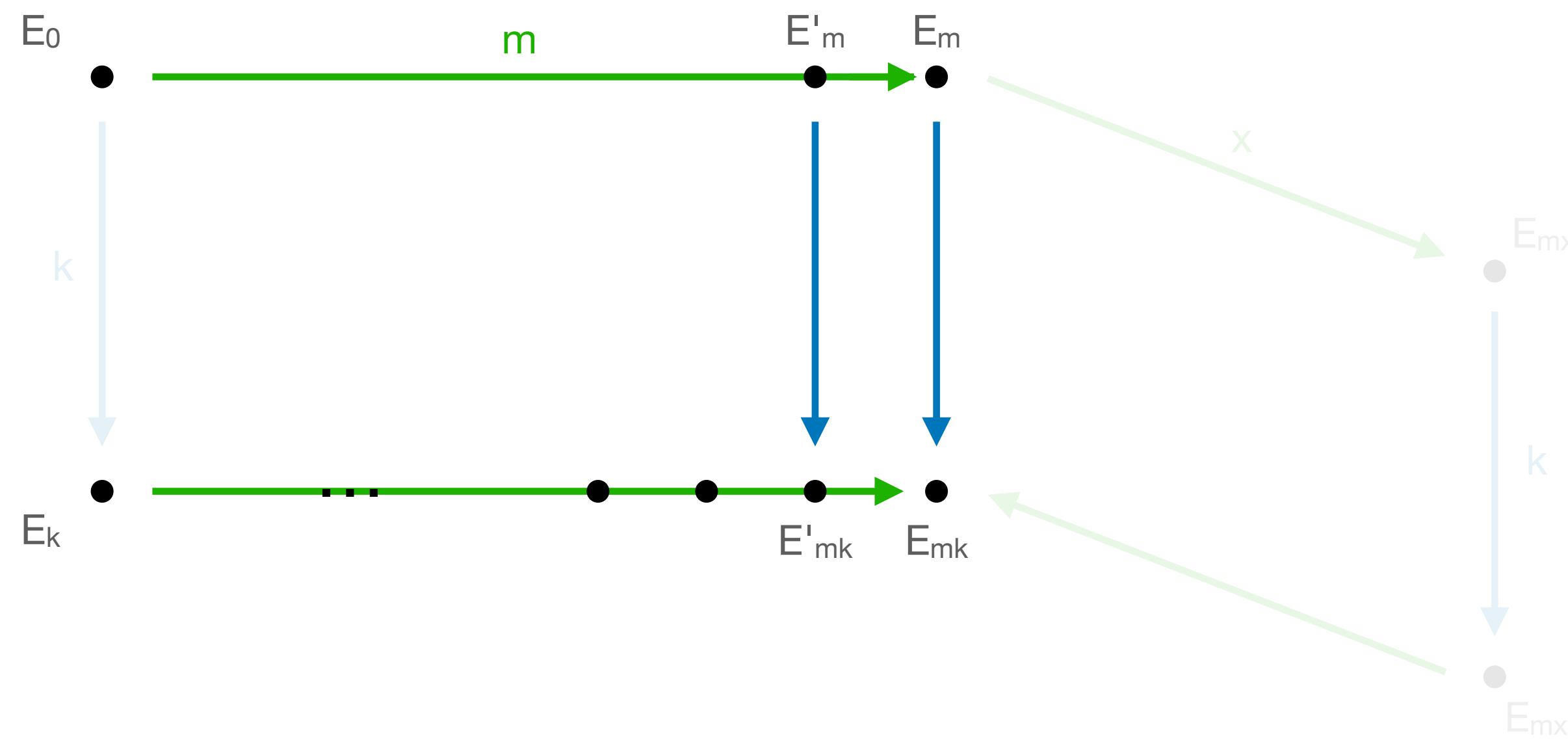
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- Repeat the attack 3 times

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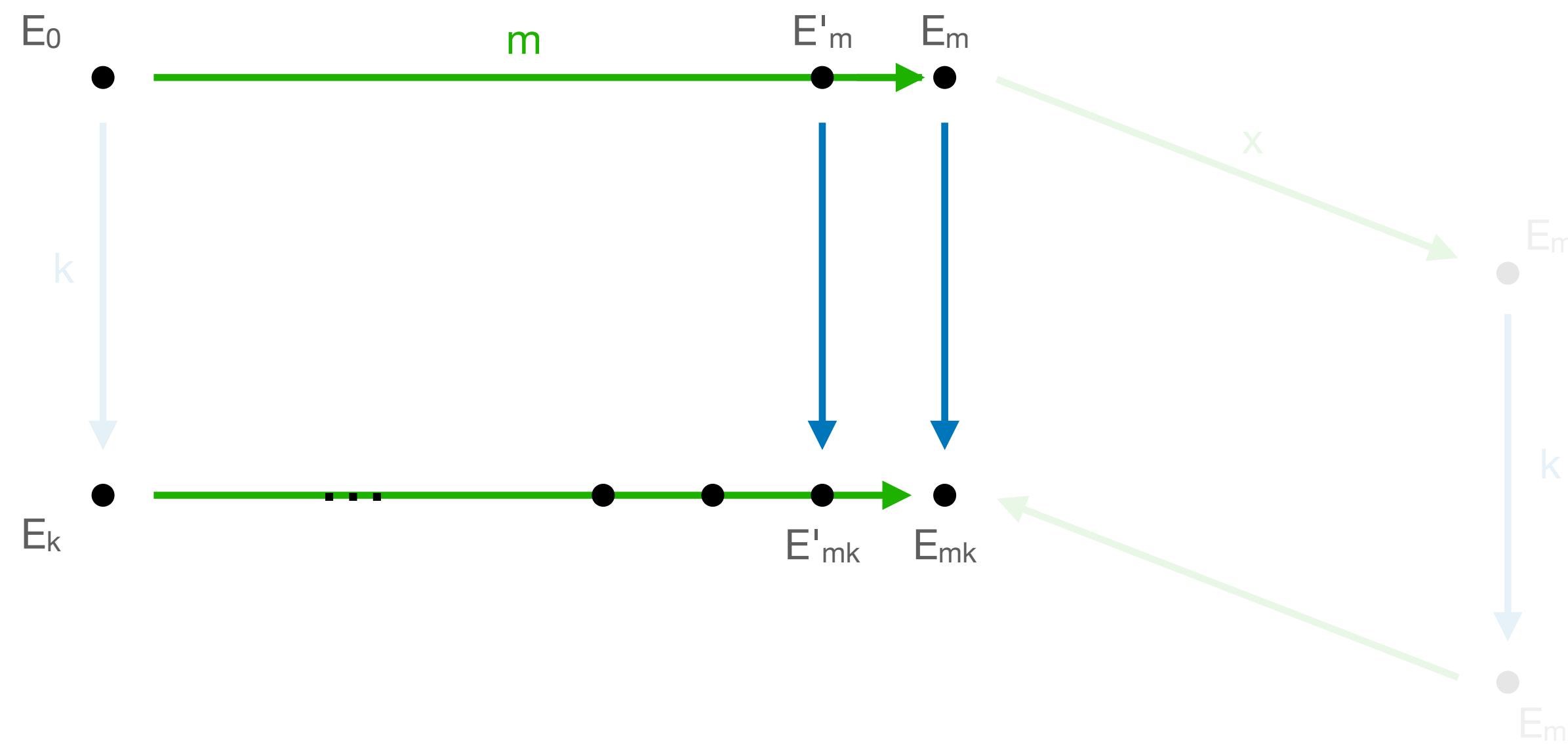
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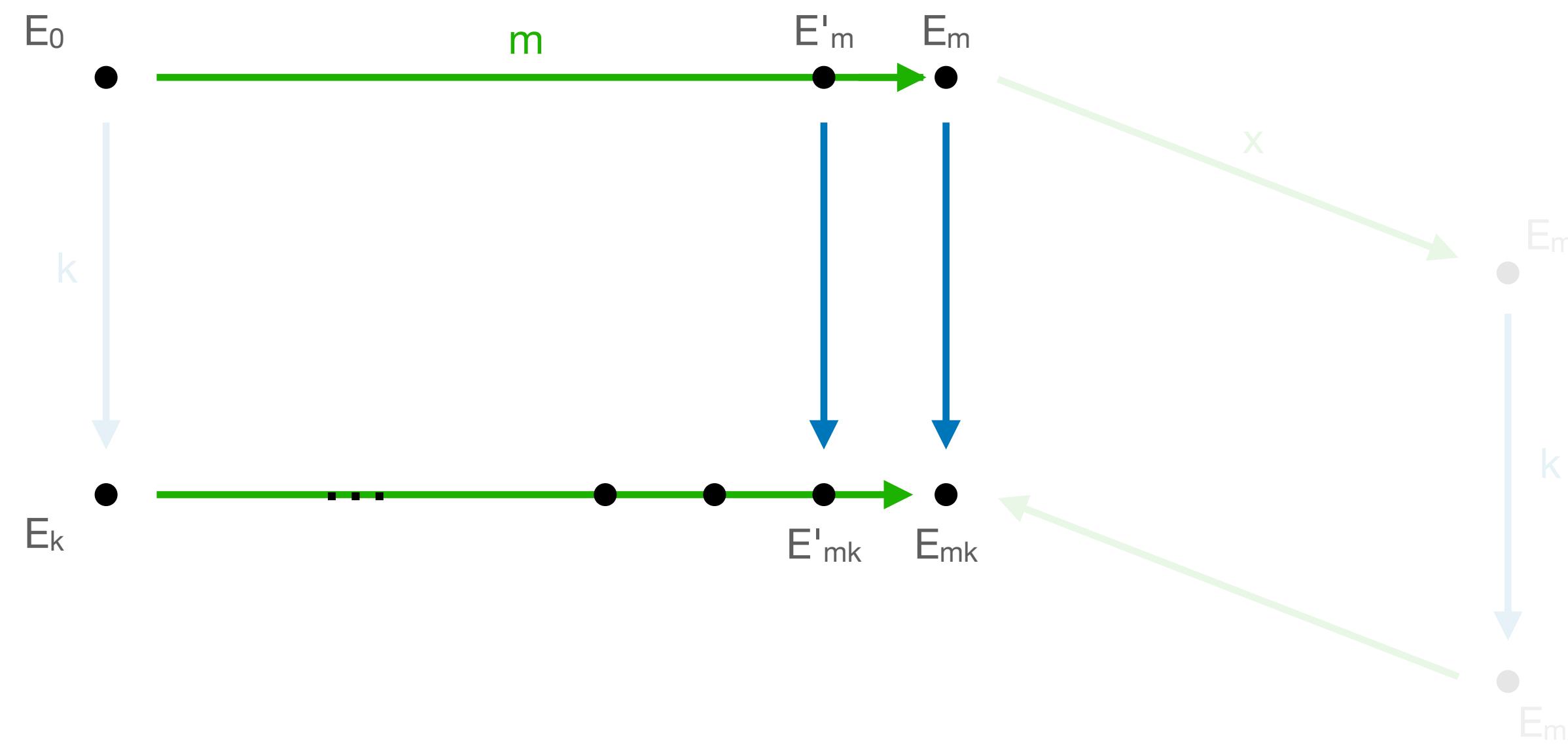
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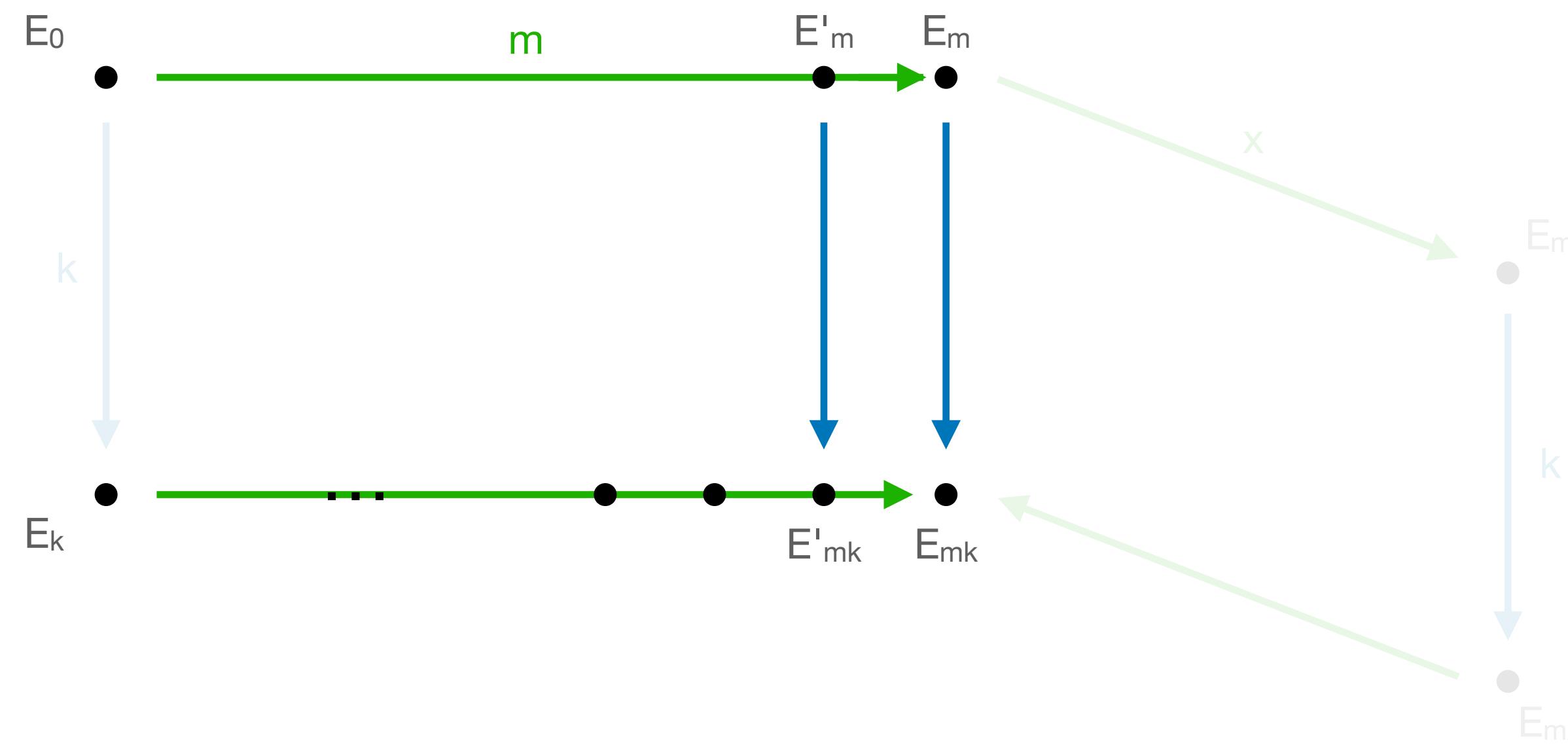
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The server can check the degree with the PoK!

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## Part 2

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Actual complexity: sub-exponential

# Countermeasures?

It seems hard to prevent an attacker from recovering a basis on  $E_k$

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## **Validate more**

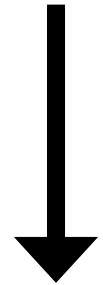
Ensure that the client submits  
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The protocol is oblivious

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## Update values

Use dynamic values for  
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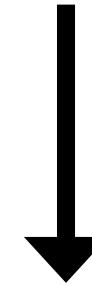
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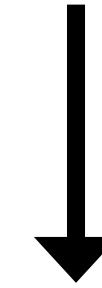
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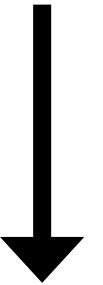
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**Idea:** make the basis on  $E_k$  not enough for an attack

# An efficient countermeasure

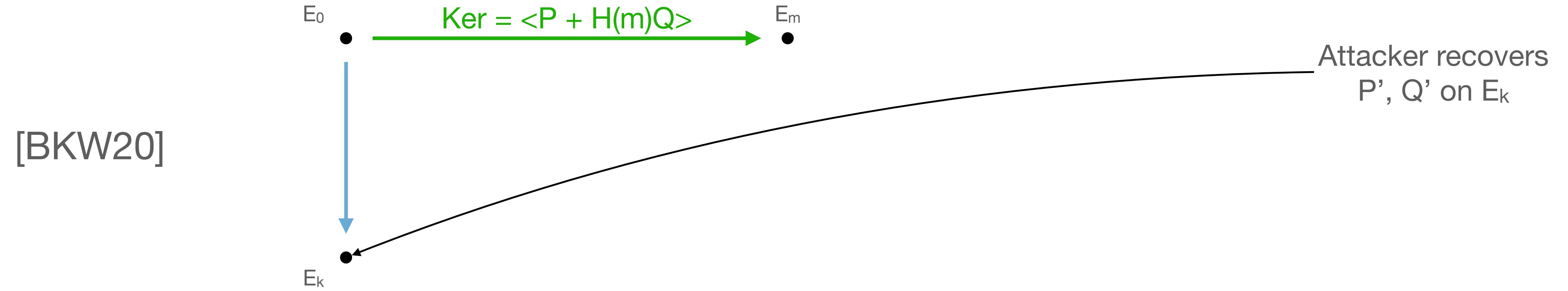
[BKW20]

# An efficient countermeasure

$$E_0 \xrightarrow{\text{Ker} = \langle P + H(m)Q \rangle} E_m$$

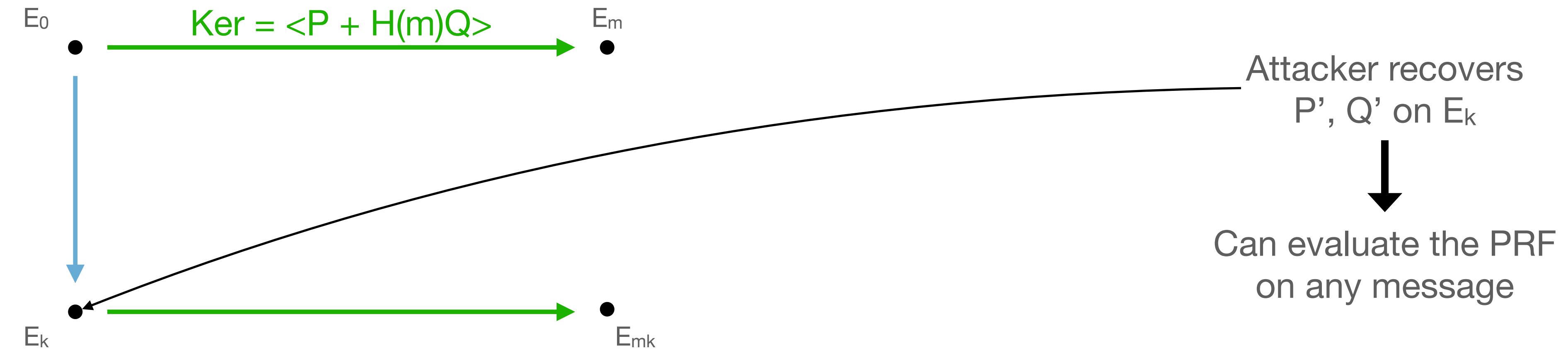
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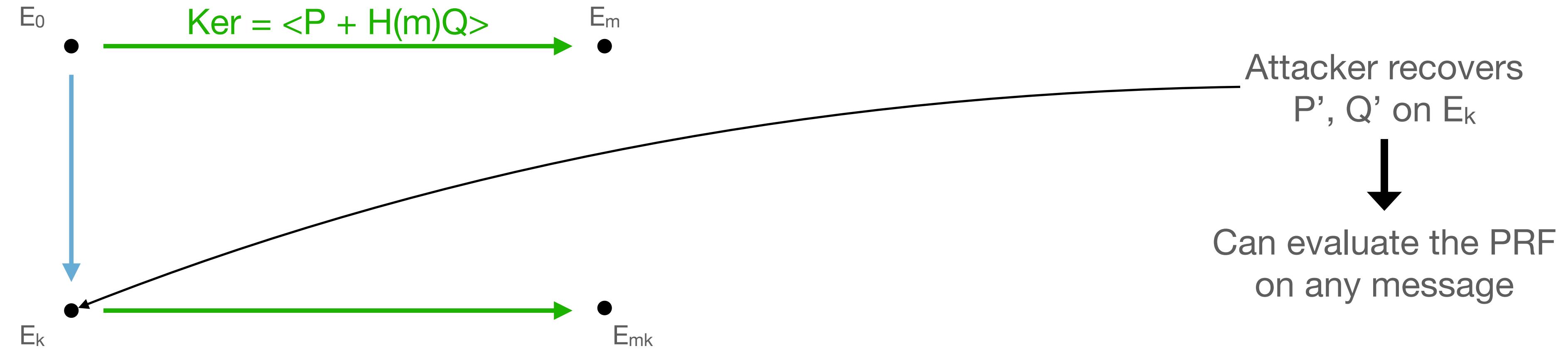
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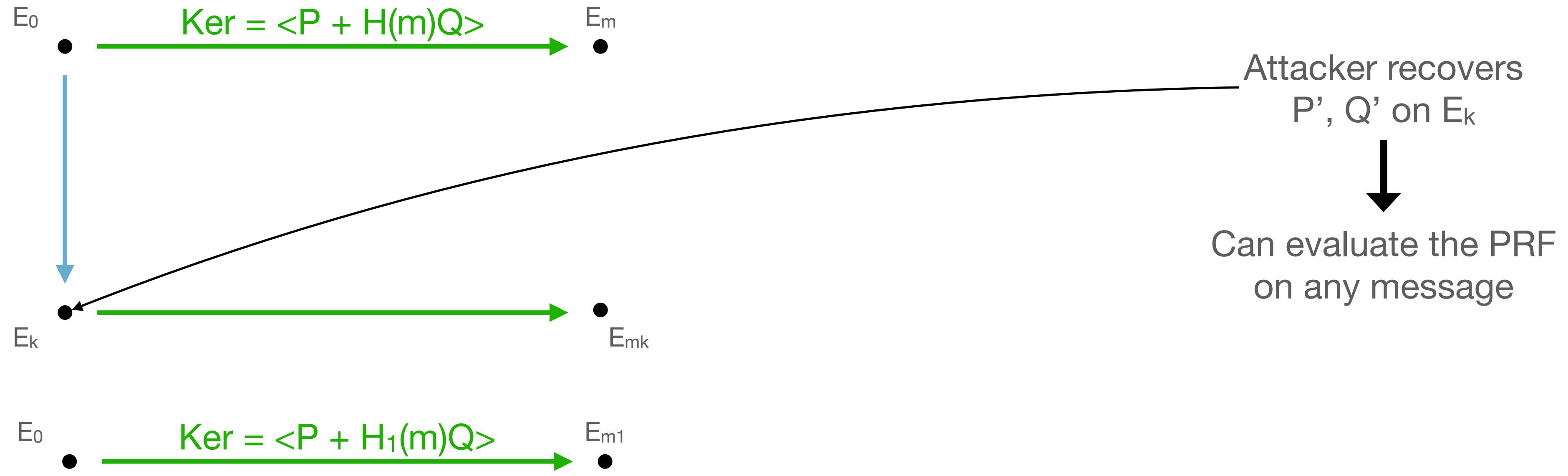
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Our  
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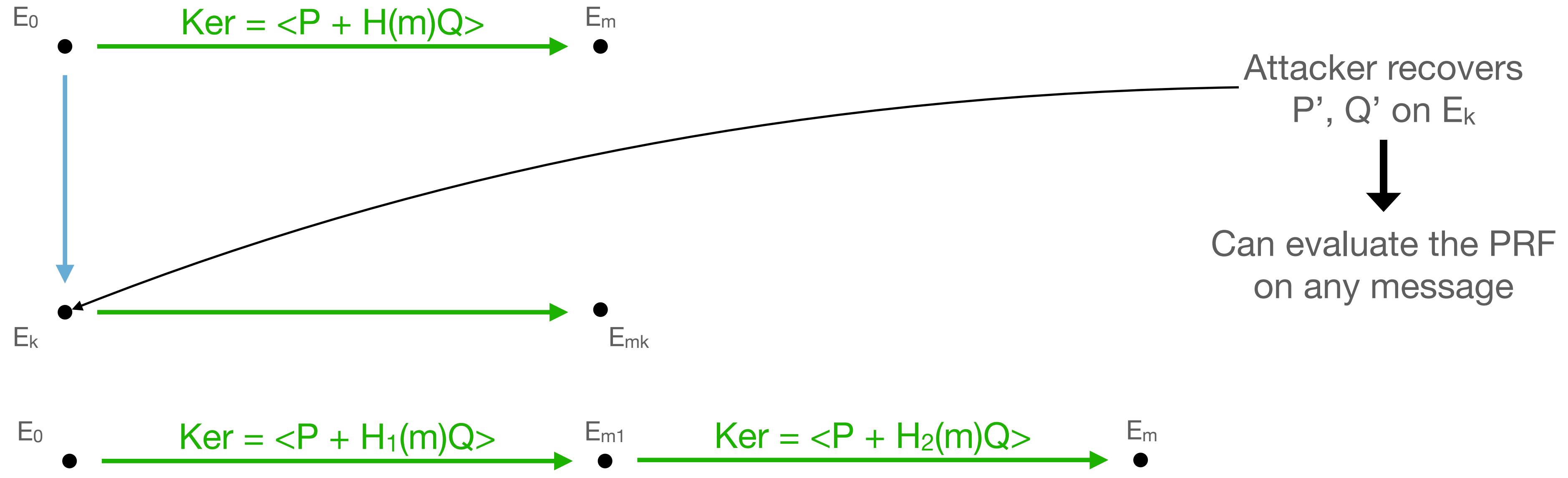
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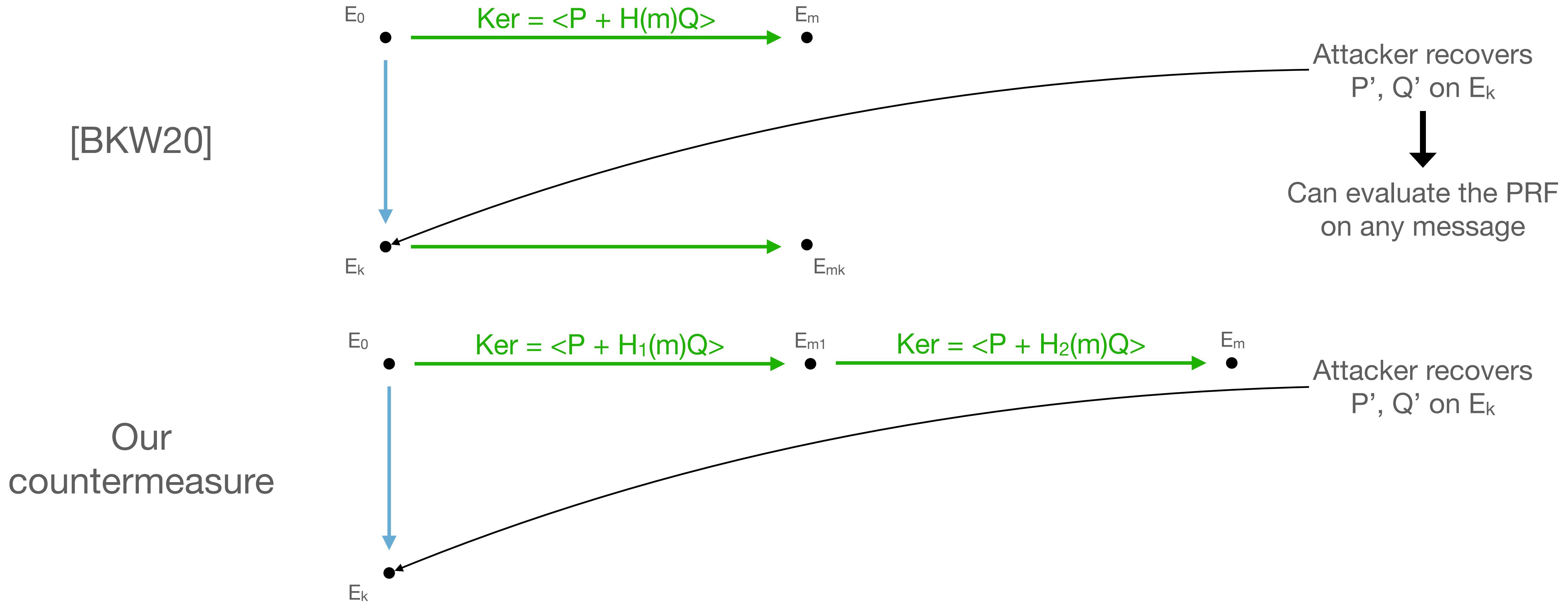
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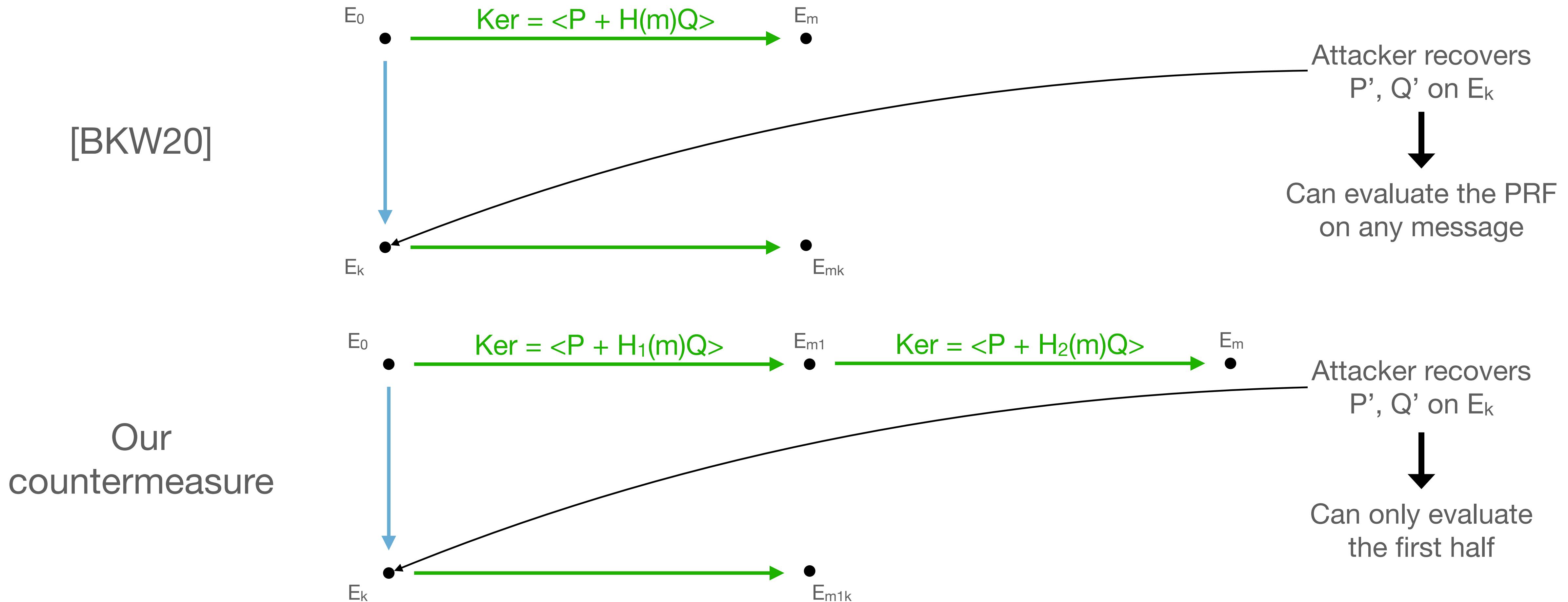


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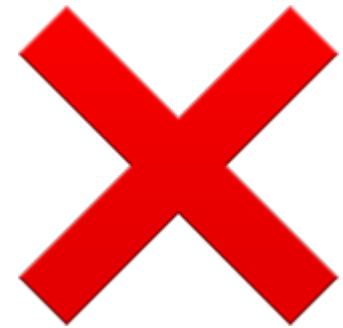
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Masked-degree isogenies  
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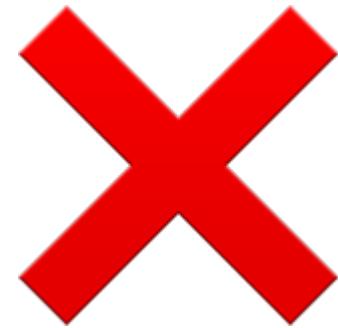
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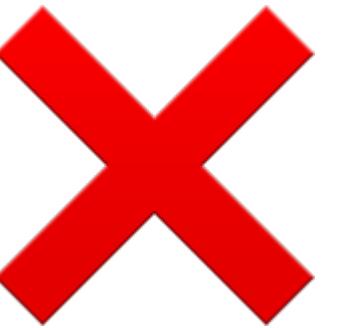
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hard to build proofs

Masked torsion points  
[Fou22,FMP23]



it works

needs new Polk

$p \approx 2^{6000}$

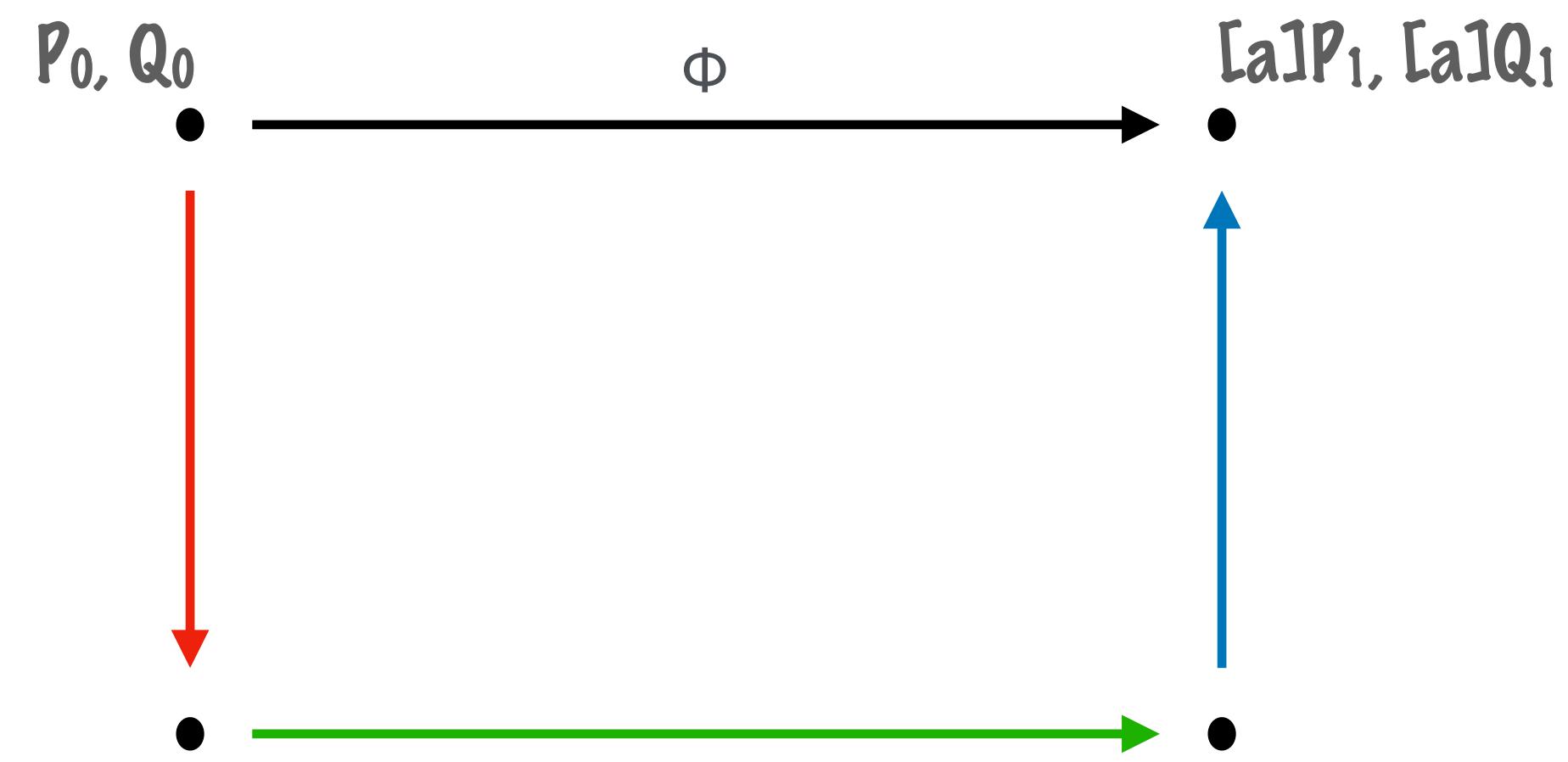
# PolK with masked torsion



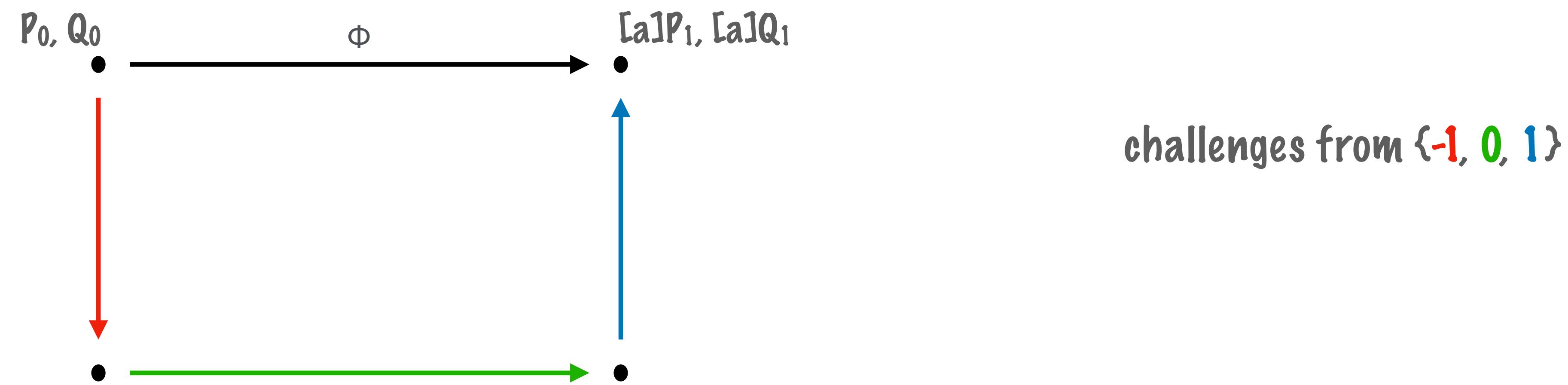
# PolK with masked torsion

$$\begin{array}{ccc} P_0, Q_0 & \xrightarrow{\phi} & [a]P_1, [a]Q_1 \\ \bullet & \longrightarrow & \bullet \end{array}$$

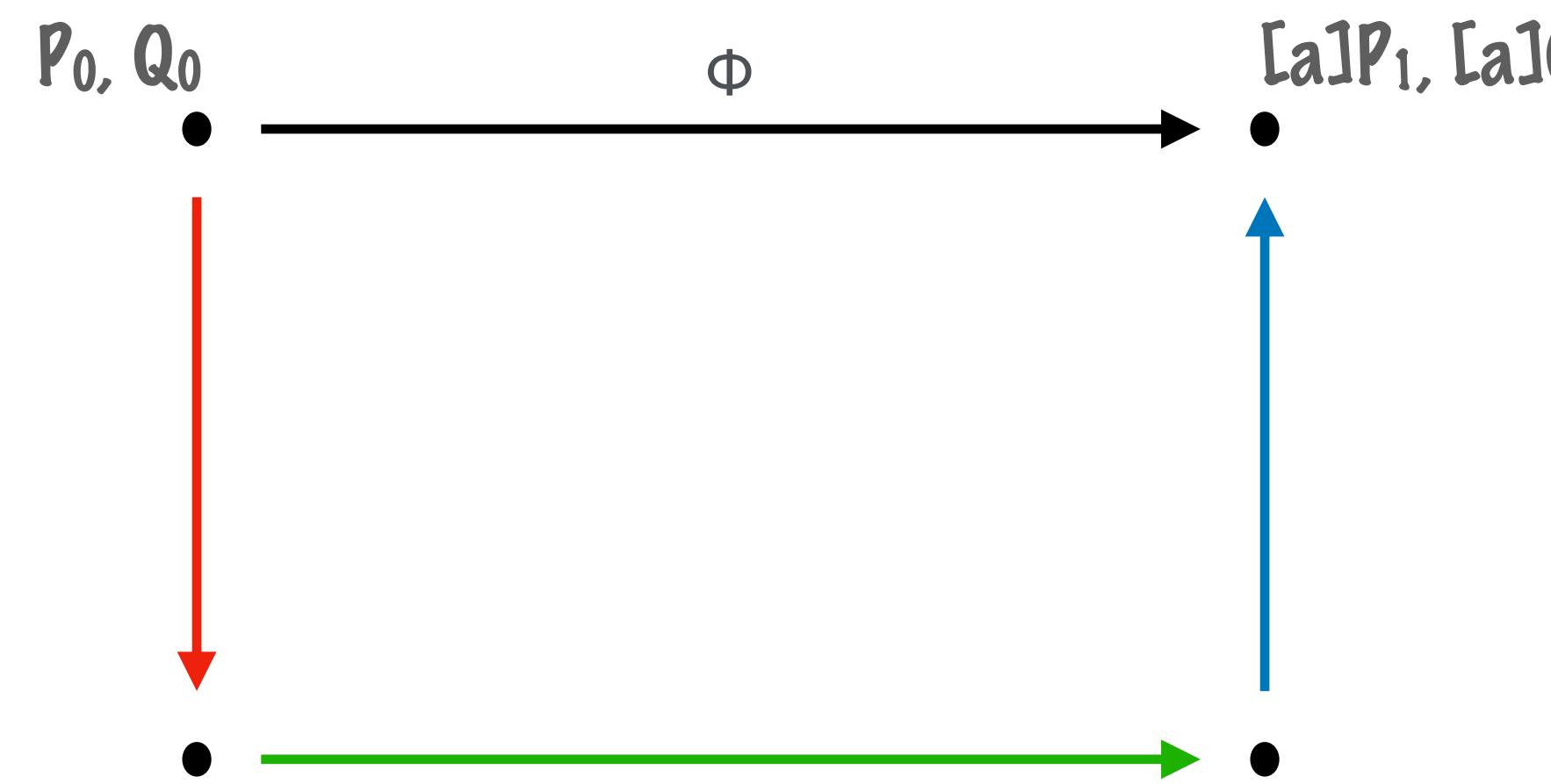
# PolK with masked torsion



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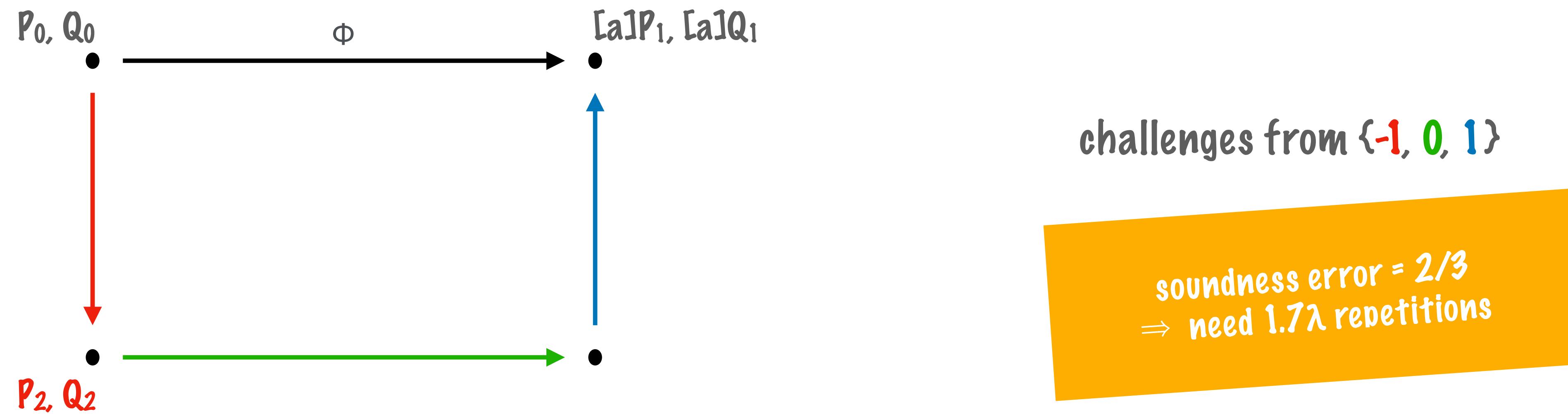
# PolK with masked torsion



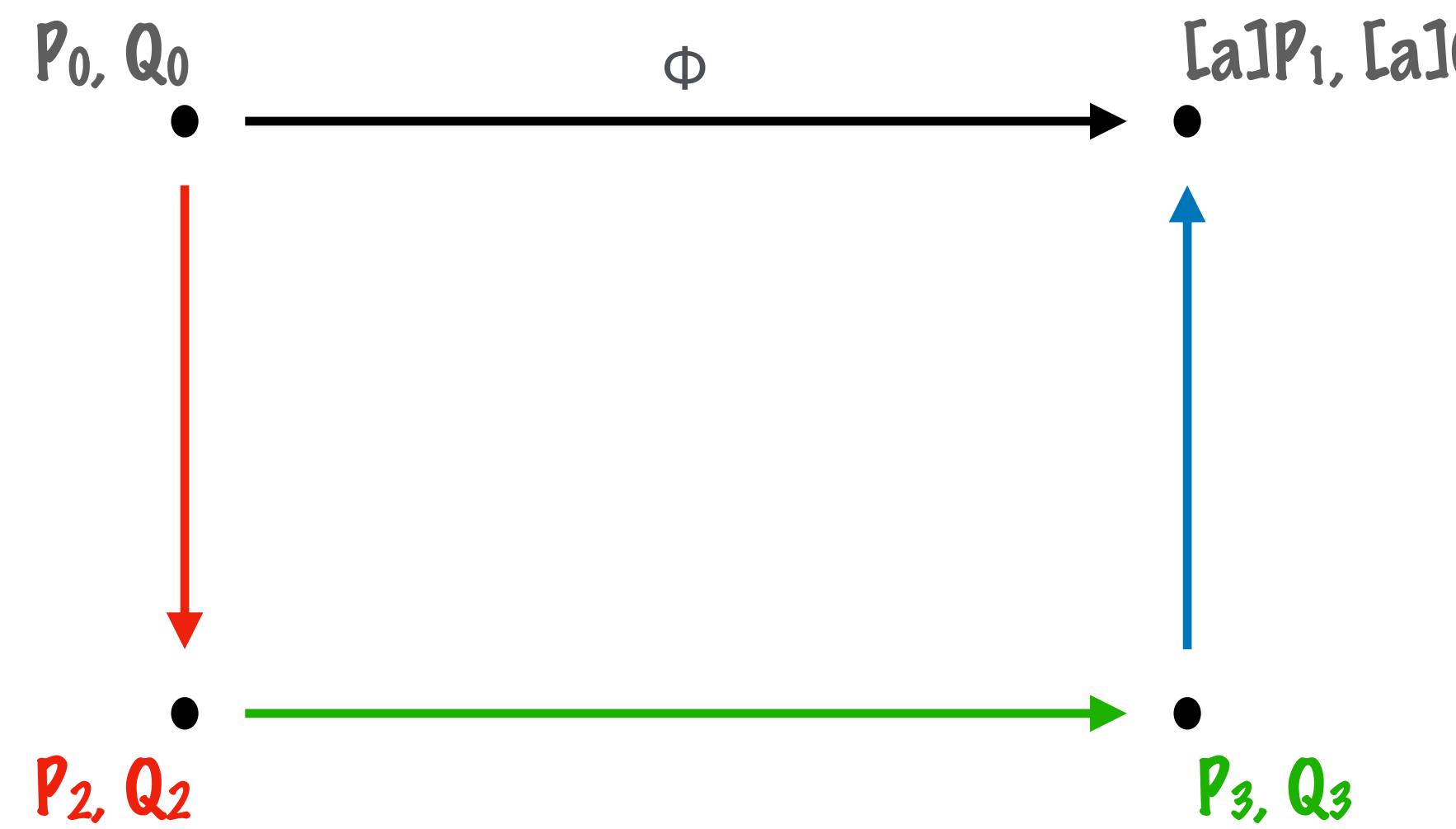
challenges from  $\{-1, 0, 1\}$

soundness error =  $2/3$   
 $\Rightarrow$  need  $1.7\lambda$  repetitions

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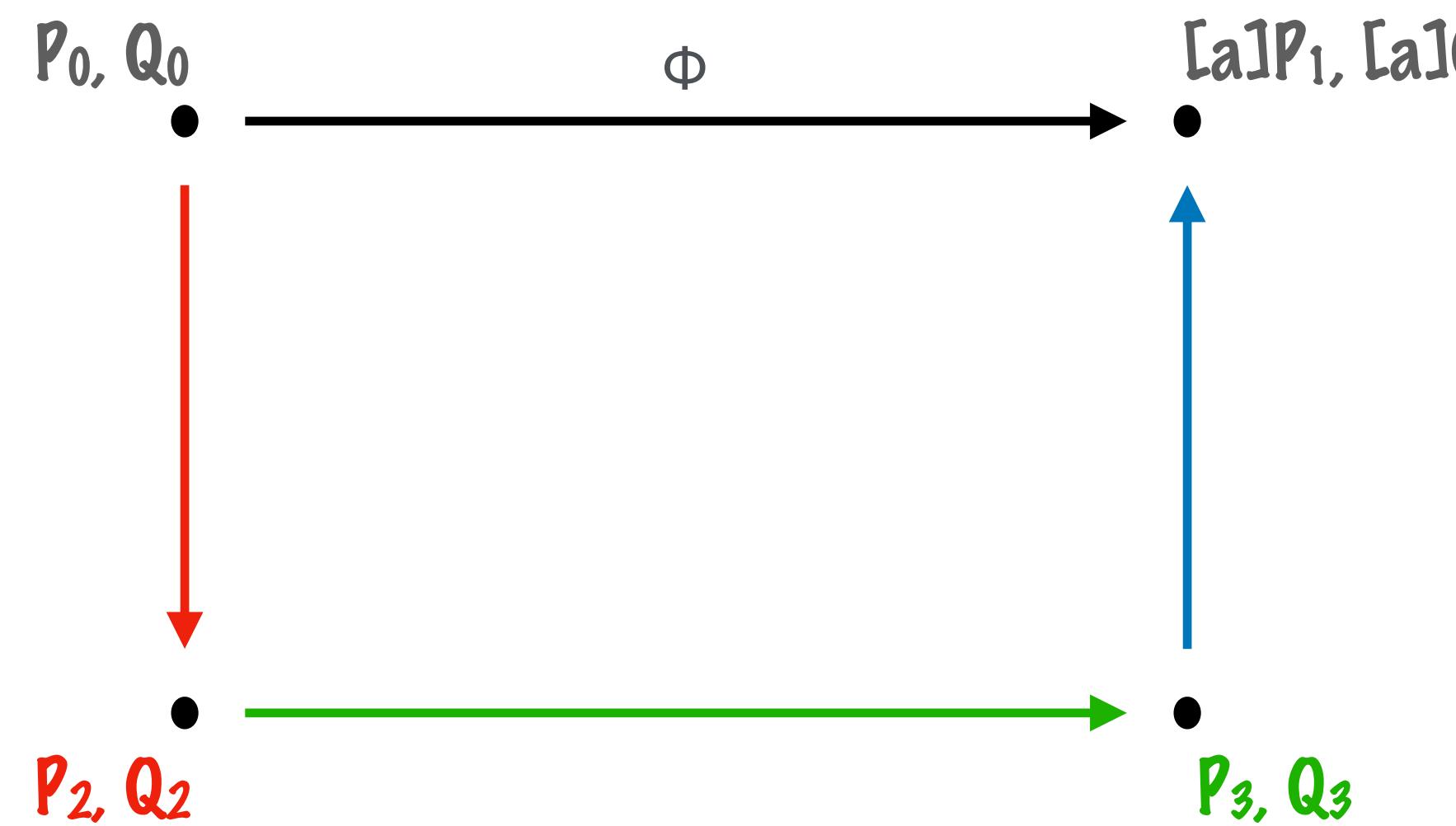
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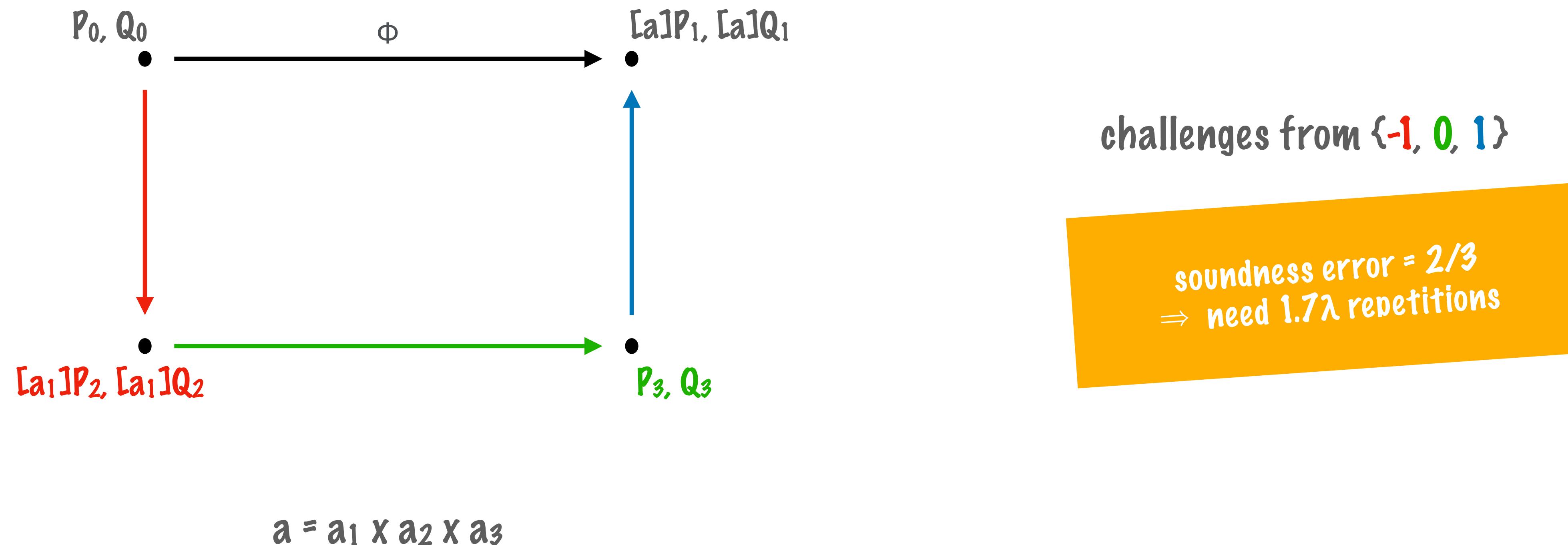


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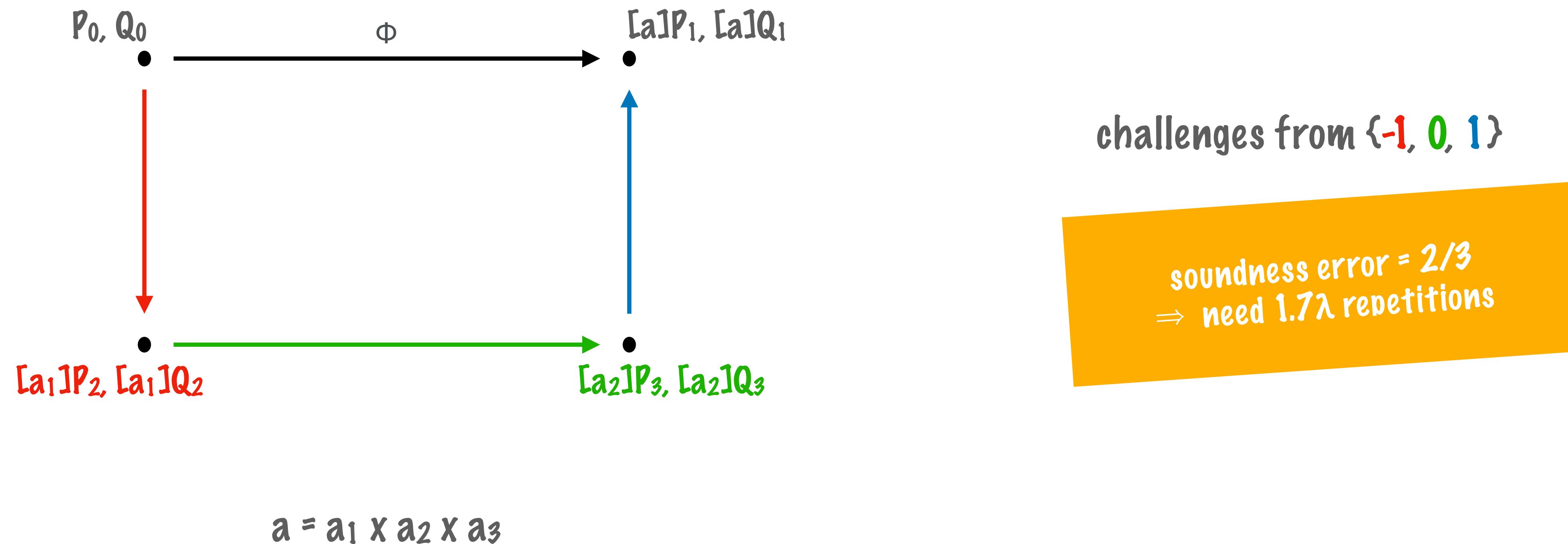
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$$a = a_1 \times a_2 \times a_3$$

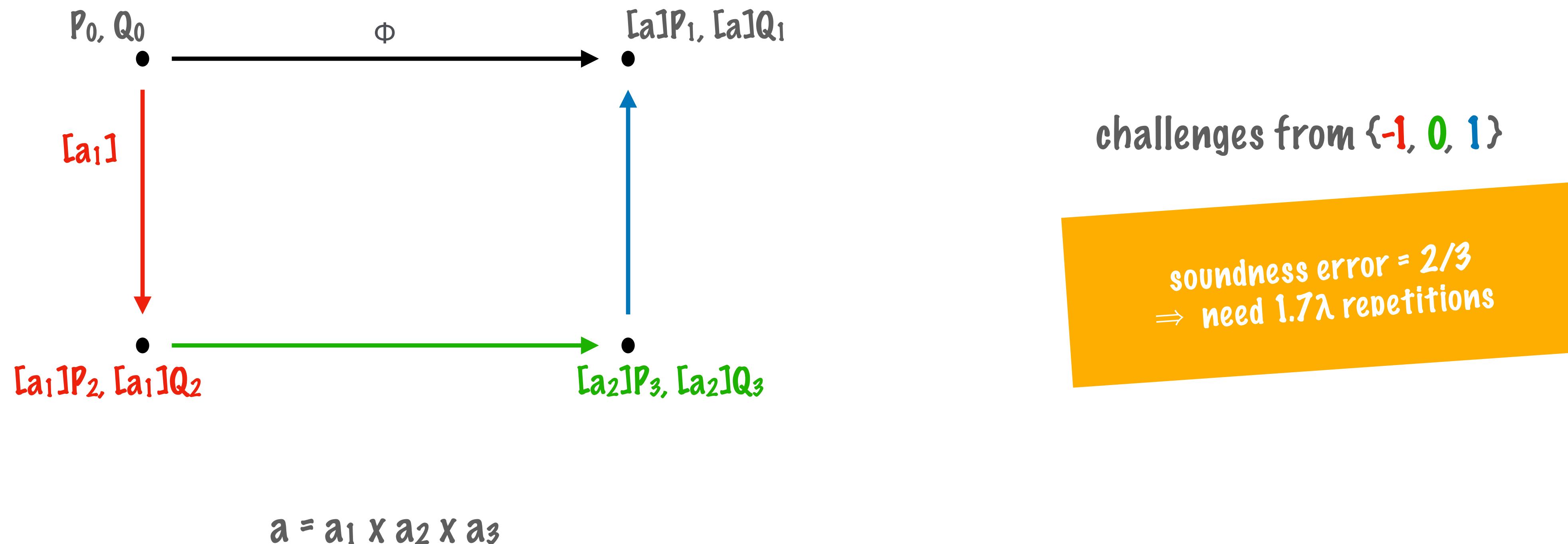
# PolK with masked torsion



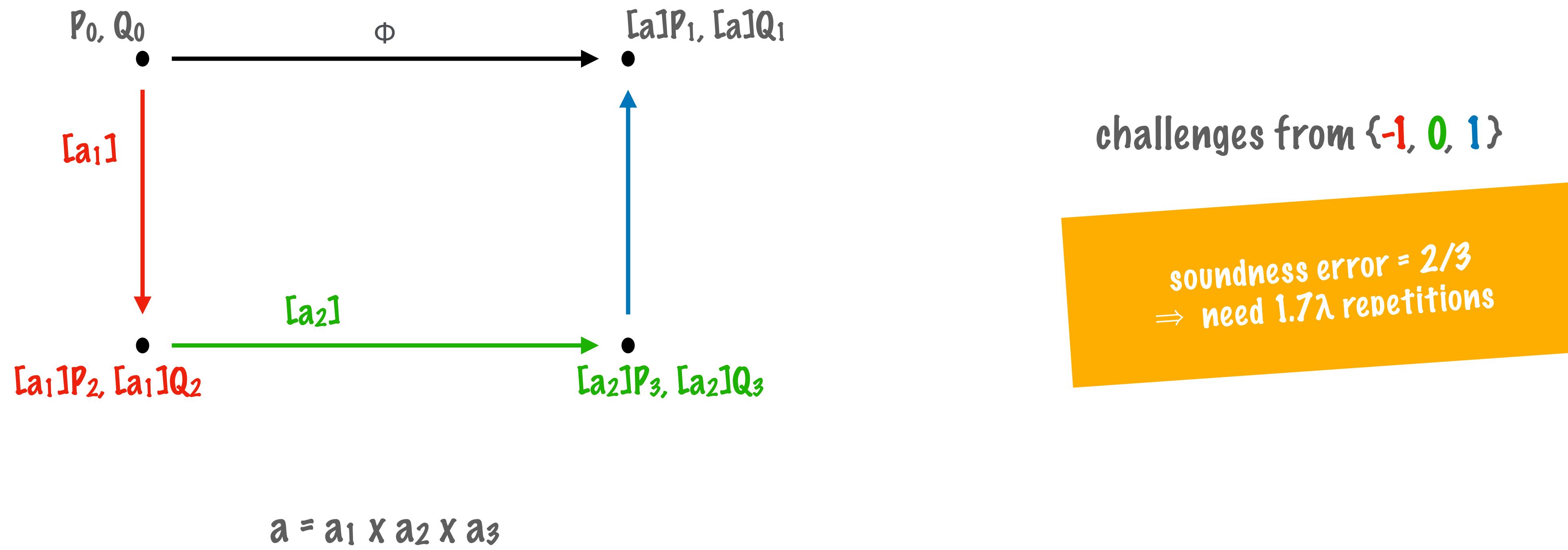
# PolK with masked torsion



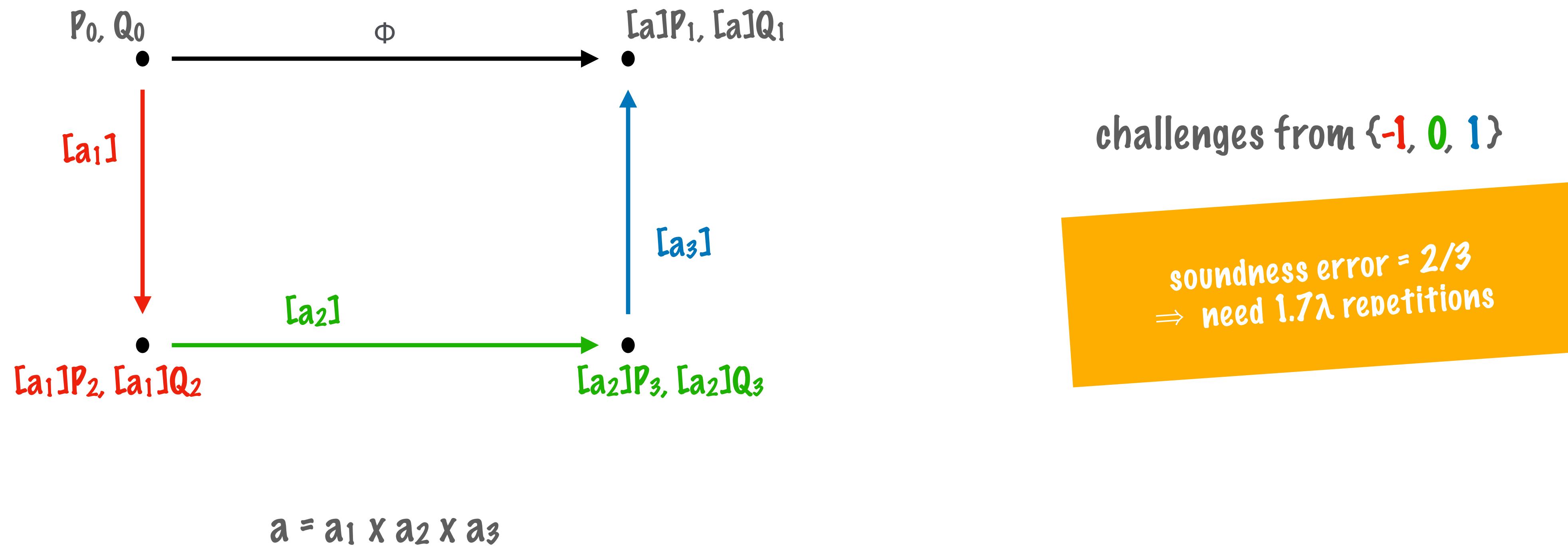
# PolK with masked torsion



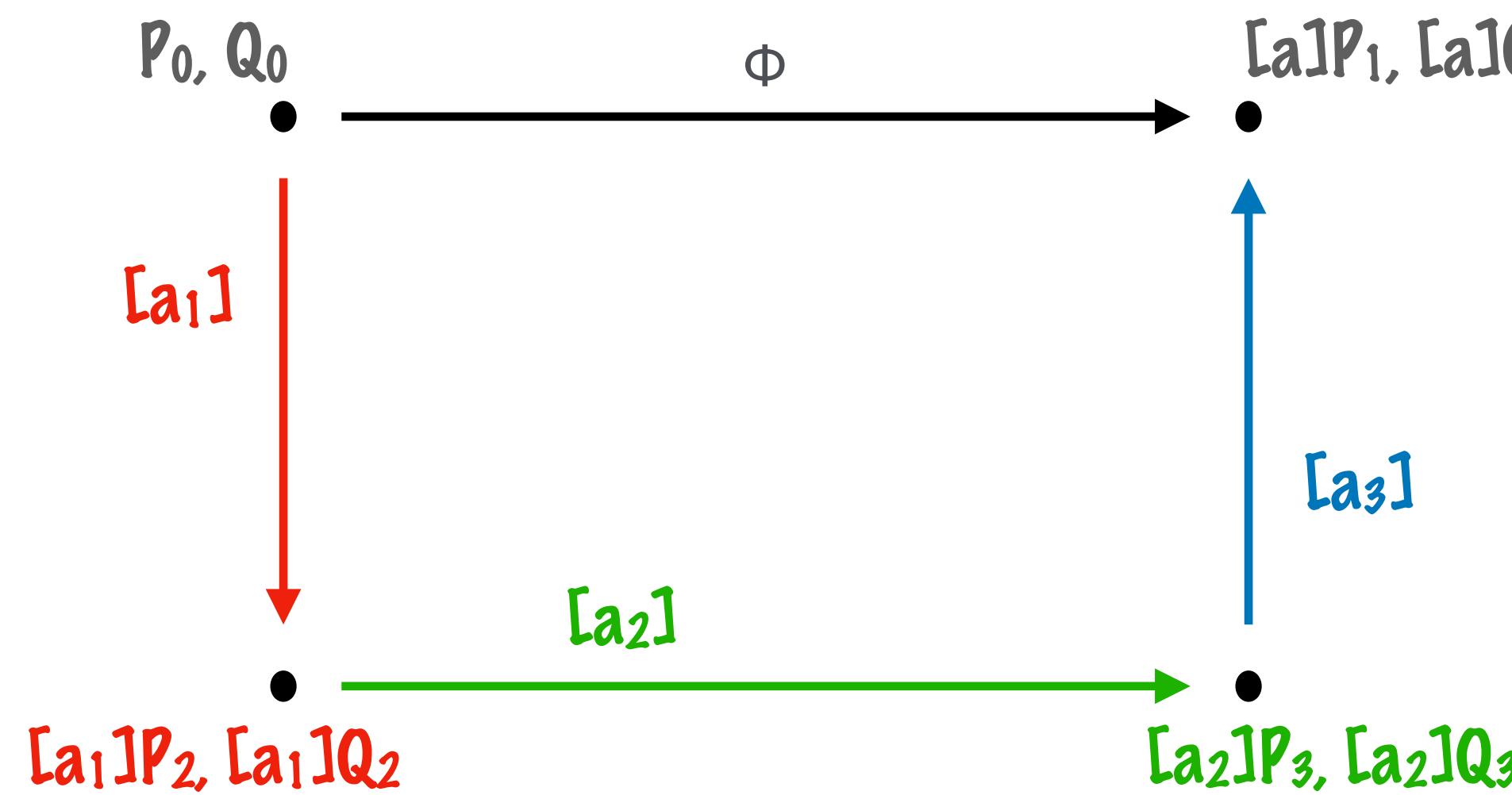
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$$a = a_1 \times a_2 \times a_3$$

$$p \approx \text{ord } P, Q \times \deg \Phi \times \deg \rightarrow \\ \approx 2^{9000}$$

# Verifiability

[BKW20] uses 3 proofs:



Server's isogeny



Server's commitment



Isogeny is parallel  
to commitment

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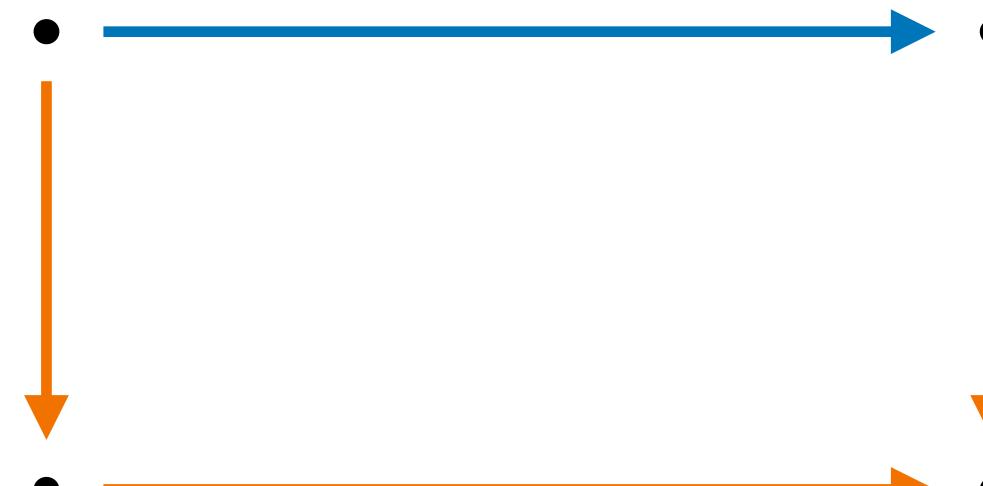


Isogeny is parallel  
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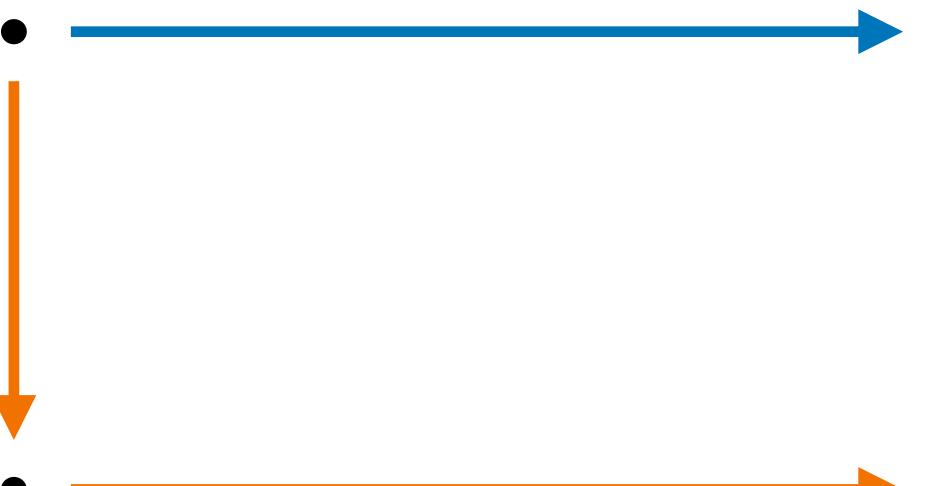
Interactive (5 rounds)

# Verifiability

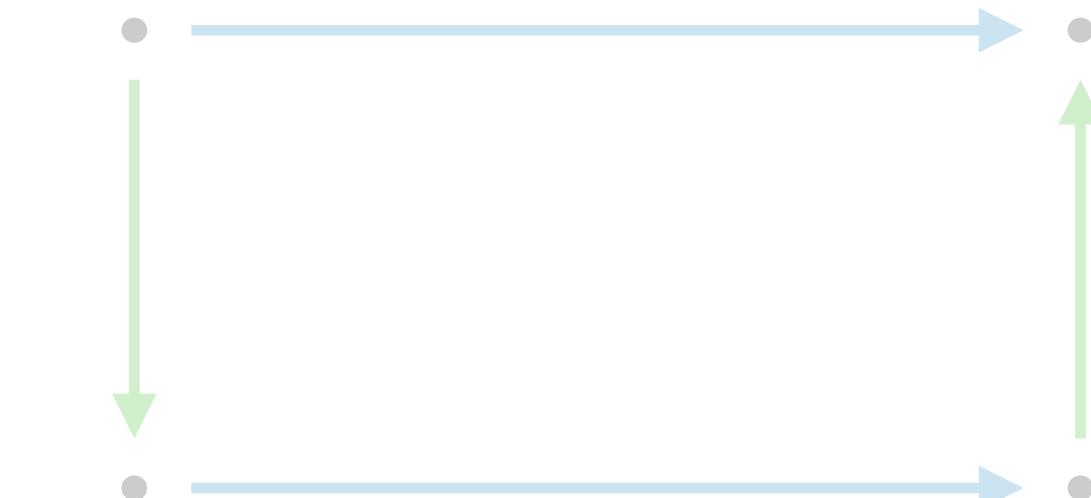
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# Verifiability

[BKW20] uses 3 proofs:



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Isogeny is parallel  
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Run together

**Interactive (5 rounds)**

# Verifiability

[BKW20] uses 3 proofs:



Server's isogeny



Server's commitment



Isogeny is parallel  
to commitment

Run together

Run together

↓  
Prove “parallelness” when  
revealing horizontal isogeny

**Interactive (5 rounds)**

# Verifiability

[BKW20] uses 3 proofs:



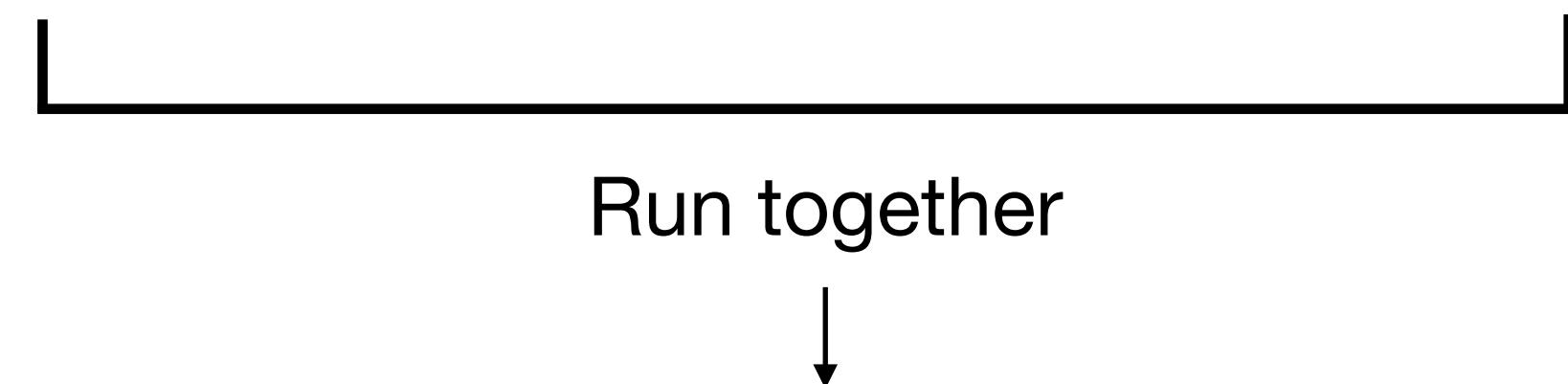
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Server's commitment



Isogeny is parallel  
to commitment



Prove "parallelness" when  
revealing horizontal isogeny

Non-interactive

Interactive (5 rounds)

# Verifiability

[BKW20] uses 3 proofs:



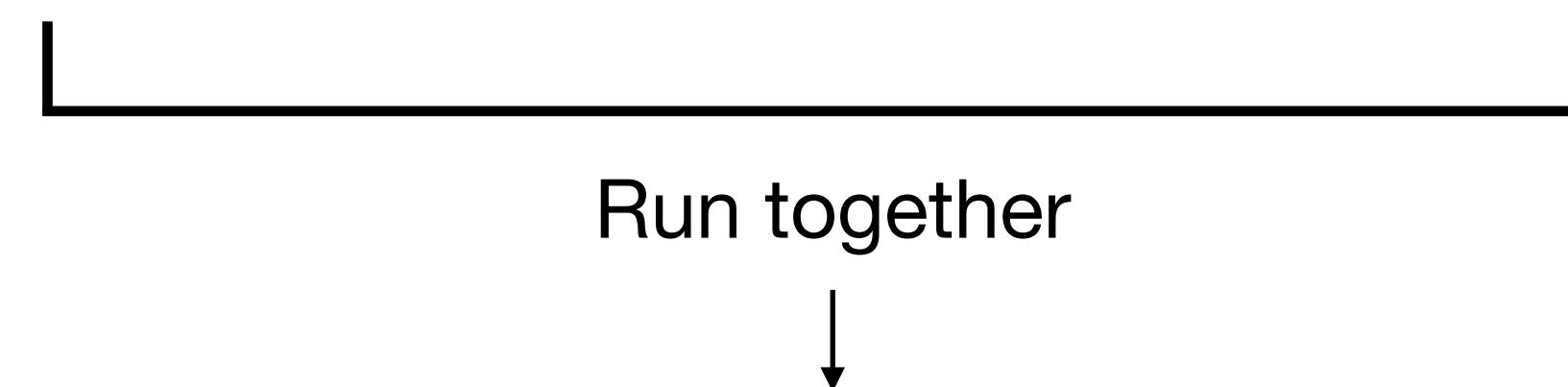
Server's isogeny



Server's commitment



Isogeny is parallel  
to commitment



Prove "parallelness" when  
revealing horizontal isogeny

Non-interactive  
Saves computations

Interactive (5 rounds)

# Putting it all together

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- One-more unpredictability countermeasure

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more efficient than original  
new security assumption

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- One-more unpredictability countermeasure
- Integrated SIDH countermeasures

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- One-more unpredictability countermeasure **more efficient than original  
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- One-more unpredictability countermeasure **more efficient than original  
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prime is still large**
- New PoPI **more efficient than original  
round optimal**

# Results

Protocol	Rounds	Bandwidth (avg.)	Verifiable	Secure
[1] (LWE)	2	>128 GB	✓	✓
[5] (CSIDH)	3	424 kB	✗	✓
[5] (SIDH) <sup>FO</sup>	6	1.4 MB	✓	✗
[5] (SIDH) <sup>Unruh</sup>	6	>10.9 MB	✓	✗
[This work] <sup>FO</sup>	2	1.9 MB	✓	✓
[This work] <sup>Unruh</sup>	2	8.7 MB	✓	✓