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Improving the Rectangle Attack on GIFT-64

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GIFT is a lightwight **bit-wise** block cipher with Substitution-Permutation-Network, which Banik *et al.* proposed at CHES' 2017.

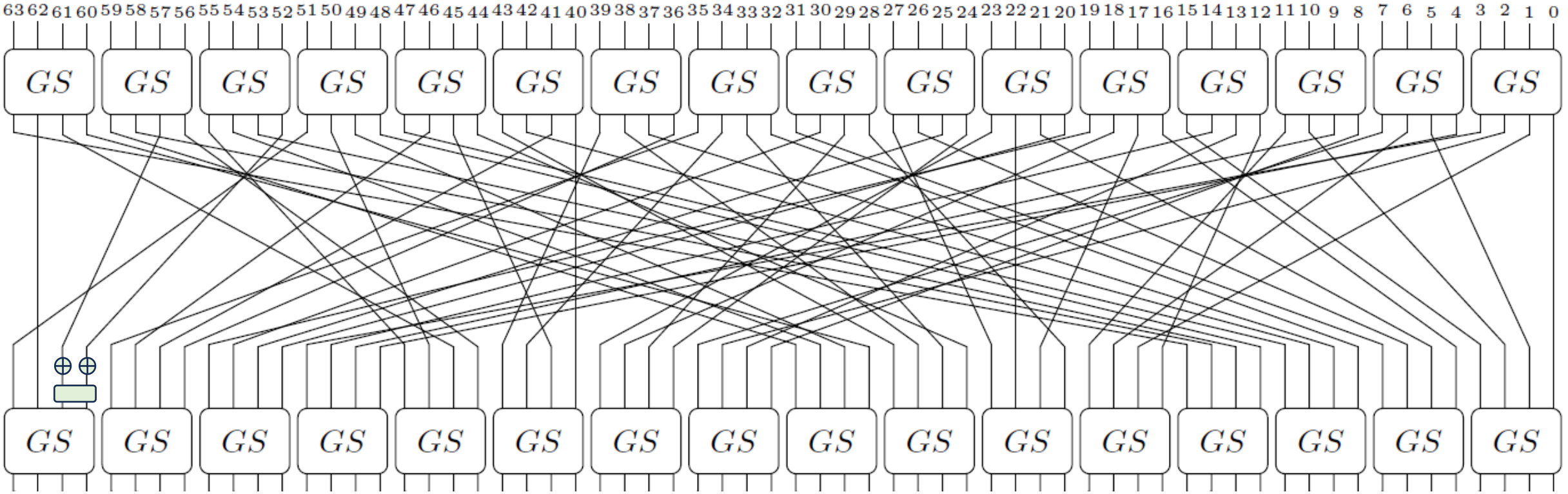

Fig. One round encryption of GIFT-64

- Bit Permutation, diffuse 4 bits in each cell to 4 cells in the next round.

$$P[i] = \left\{4\lfloor\frac{i}{16}\rfloor + 16\left(3\lfloor\frac{i \bmod 16}{4}\rfloor + (i \bmod 4) \bmod 4\right) + (i \bmod 4)\right\} \bmod 64.$$

- Key Schedule, rotation only, reused in 5 rounds.

$$k_7||k_6 \ldots||k_1||k_0\ (16\ bits * 8) \leftarrow K\ (128\ bits)$$

Subkey update

$$k_7||k_6|| \ldots ||\ k_1\ ||k_0 \leftarrow\ k_1 \ggg 2||k_0\ \ggg 12\ ||\ \ldots||k_3||k_2$$

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- The Differential attack [Biham *et al.* in 1991]:
  find $\Delta P \rightarrow \Delta C$ with high probability P (Hard to find long trails).

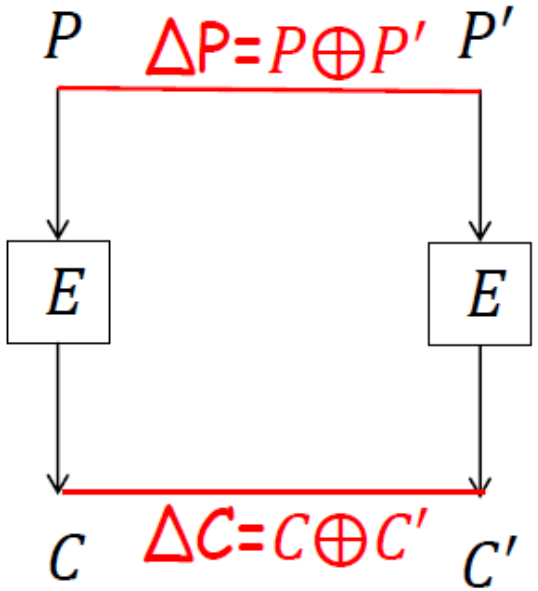

Fig. Differential

$$P = P_r\{C \oplus C' = \Delta C | P \oplus P' = \Delta P\}$$

- The Boomerang attack [Wagner in 1999]:

  Treat cipher E as $E_0 \circ E_1$, calculate the probability (p and q) and connect them.

◆Adaptive chosen plaintext/ciphertext attack.

$$\begin{cases} \alpha \to \beta & with\ probability\ \boldsymbol{p} \\ \gamma \to \delta & with\ probability\ \boldsymbol{q} \end{cases}$$

$$P_{boomerang} = \boldsymbol{p^2 q^2}$$

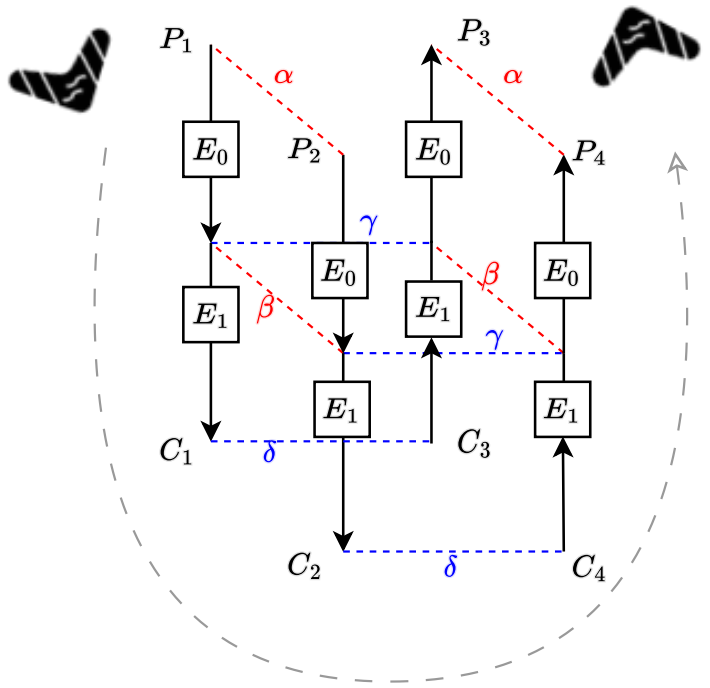

Fig. Boomerang attack

Boomerang attack

Amplified boomerang attack

[Kelsey *et al.* in 2000]

(choose plaintext/ciphertext)

Rectangle attack

[Biham *et al.* in 2001]

(choose plaintext/ciphertext)

$$\hat{p} = \sqrt{\sum_i P_r^2(\alpha \to \beta_i)}$$

$$\hat{q} = \sqrt{\sum_j P_r^2(\gamma_i \to \delta)}$$

$$P_{rectangle} = \mathbf{2^{-n}\hat{p}^2\hat{q}^2}$$

◆Steps:

- Choose plaintext $P_1 \oplus P_2 = P_3 \oplus P_4 = \alpha$.

$$P_1, P_2, P_3, P_4 \xrightarrow{E_k} C_1, C_2, C_3, C_4.$$

- If $C_1 \oplus C_3 = C_2 \oplus C_4 = \delta$, right quartet

$$\{(P_1, C_1), (P_2, C_2), (P_3, C_3), (P_4, C_4)\}.$$

- Song *et al.* proposed the most efficient and generic rectangle key recovery algorithm at ASIACRYPT 2022

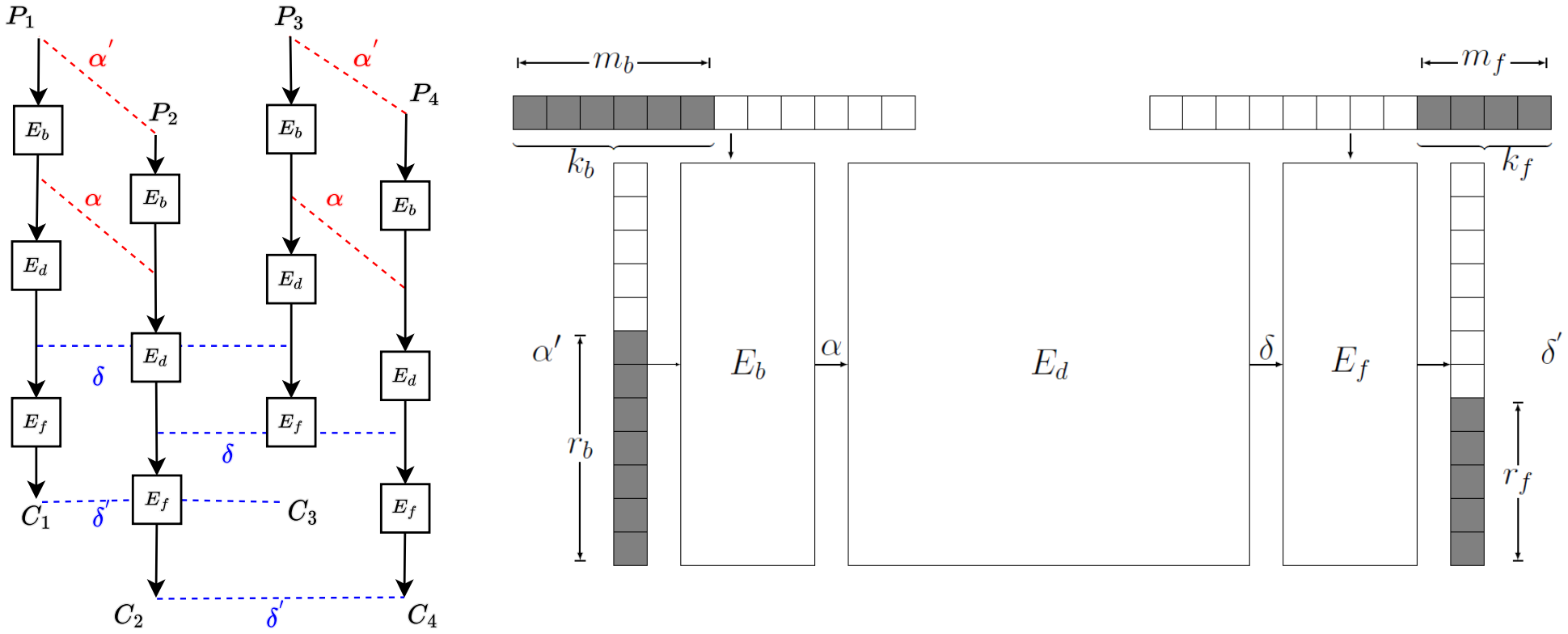

Fig. Outline of rectangle key recovery attack

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◆ Notation

$$\Delta Plaintext \xrightarrow{GS\ \&\ Bit\ permutation} \Delta Z_1 \xrightarrow{\boldsymbol{k_1}||\boldsymbol{k_0}} \Delta X_2 \xrightarrow{GS} \Delta Y_2 \cdots\cdots$$

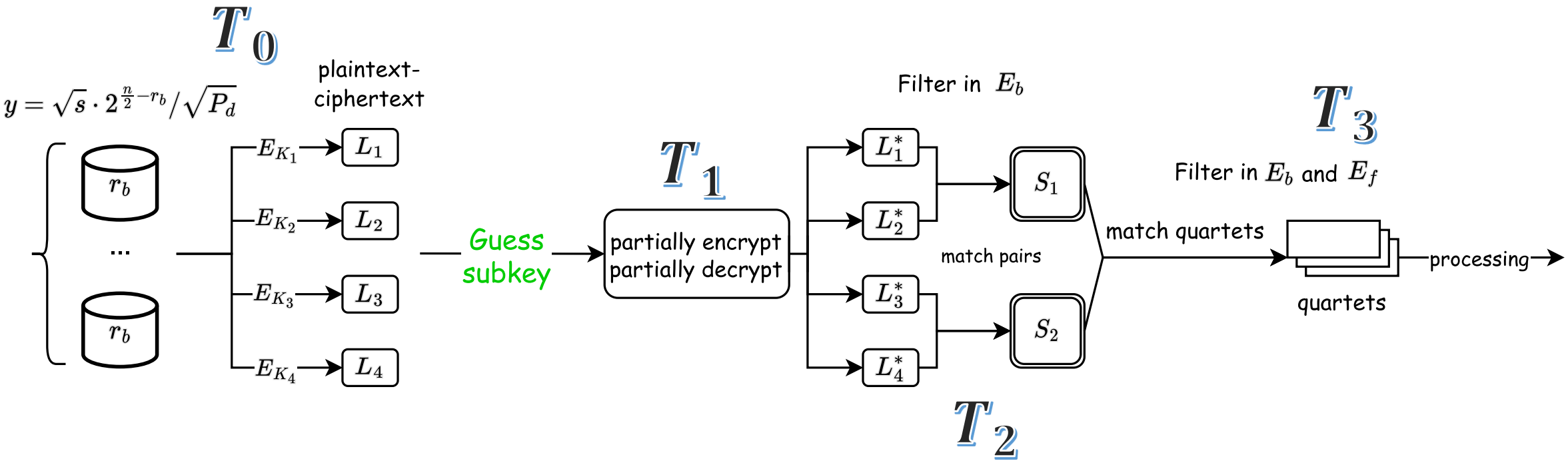
- **Based on the 20-round** rectangle distinguisher of GIFT-64 which proposed by Ji et al. in SAC 2020
- Extend **3** rounds forward and backward from the distinguisher
- The state: (backward i.e. E_b)

  $\boldsymbol{\Delta X}$: after ***subkey addition*** ⇔ before ***Sbox***;

  $\boldsymbol{\Delta Y}$: after ***Sbox*** ⇔ before ***bit permutation***;

  $\boldsymbol{\Delta Z}$: after ***bit permutation*** ⇔ before ***subkey addition***.

- The related-key rectangle key recovery attack on GIFT-64.



- Exhaustive search T_4

➢ Core idea:

Tradeoff the time complexity of each attack phase.

➢ Main question:

How to guess subkey bit and obtain the filtering bits?

➢ Basic rules:

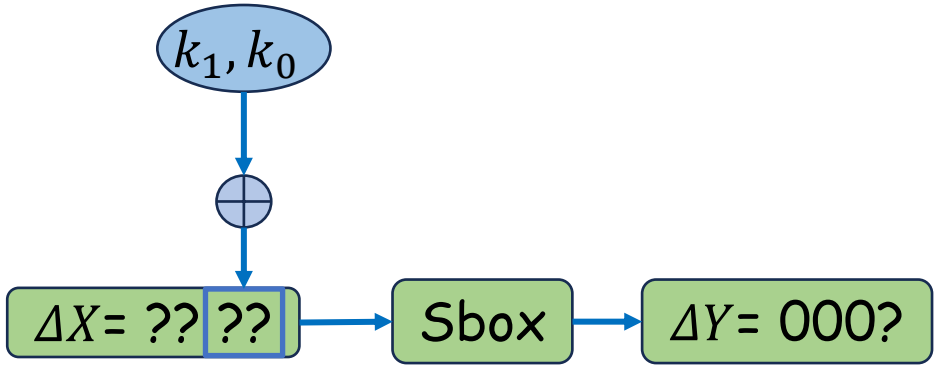
- ◆ Guessing fewer, acquiring more
- ◆ Balance the time complexity of
  - partially encryption and decryption. (T_1)
  - pair generation (T_2)
  - quartet generation and processing (T_3)

◆ How to get filtering bits by guessing subkey bits?

➢ Guessing m'_b $(resp. m'_f)$ subkey bits to obtain > $\mathbf{m'_b}$ **filtering bits** r'_b $(resp. r'_f)$

- Example:

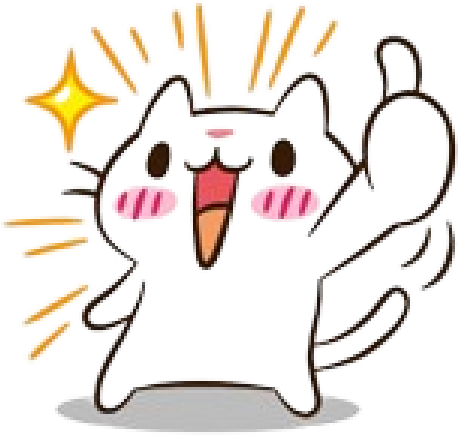
  3 bit fix diff. (000?) - 0 bit fix diff. (????) = get 3 filtering bits



➢ Advantage for time complexity: $\mathbf{2^2 \times 2^{-3} = 2^{-1}}$

More advantage with subkey reusing:

- Highlight green subkey bits can be used in round **1** and round **25**
- Highlight cyan subkey bits can be used in round **2** and round **26**
- Maximize $\frac{\textbf{\textit{guessed subkey bits}}}{\textbf{\textit{filtering bits}}}$ ⇔ Maximize advantage !!



ΔZ_1	????	????	????	????	0000	0000	0000	0000	11??	????	????	????	????	11??	????	????
$k_1 \| k_0$	15|15	14|14	13|13	12|12	11|11	10|10	9|9	8|8	7|7	6|6	5|5	4|4	3|3	2|2	1|1	0|0
ΔZ_2	????	0000	?1??	0000	0000	0000	0000	0000	0001	0000	0000	0000	0000	0000	0000	?1??
$k_3 \| k_2$	15|15	14|14	13|13	12|12	11|11	10|10	9|9	8|8	7|7	6|6	5|5	4|4	3|3	2|2	1|1	0|0
...	... ...															
ΔZ_{25}	??0?	??0?	??0?	??0?	???0	???0	???0	???0	0???	0???	0???	0???	?0??	?0??	?0??	?0??
$k_1 \| k_0$	11|7	10|6	9|5	8|4	7|3	6|2	5|1	4|0	3|15	2|14	1|13	0|12	15|11	14|10	13|9	12|8
ΔZ_{26}	????	????	????	????	????	????	????	????	????	????	????	????	????	????	????	????
$k_3 \| k_2$	11|7	10|6	9|5	8|4	7|3	6|2	5|1	4|0	3|15	2|14	1|13	0|12	15|11	14|10	13|9	12|8

Table. reused subkey bits

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◆ Variables:

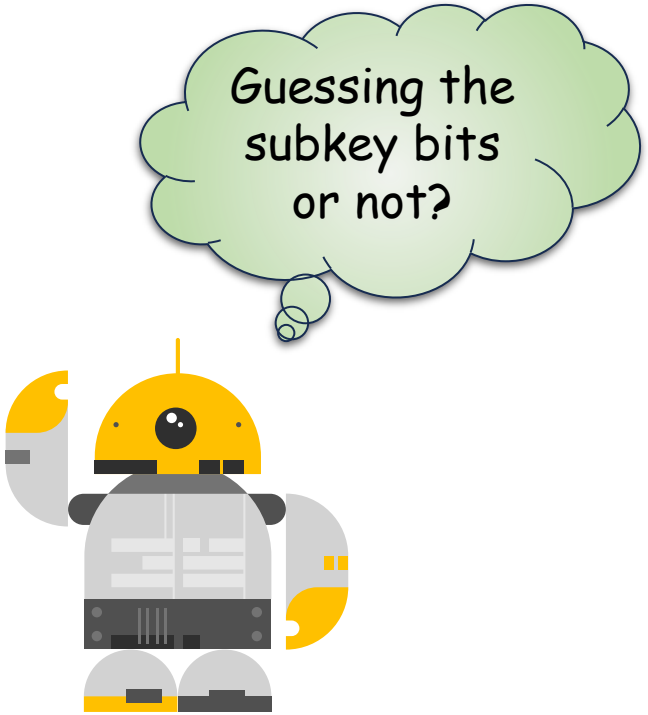
- All involved subkey bits;
- All difference of involved state bits (fixed or unknown).

◆ Constraints

- The relation between guessing subkey bits and fixed state bits;
- The relation between {guessed subkey bits, fixed state bits} and time complexity.

◆ Objective function

- **The most balanced time complexity.**



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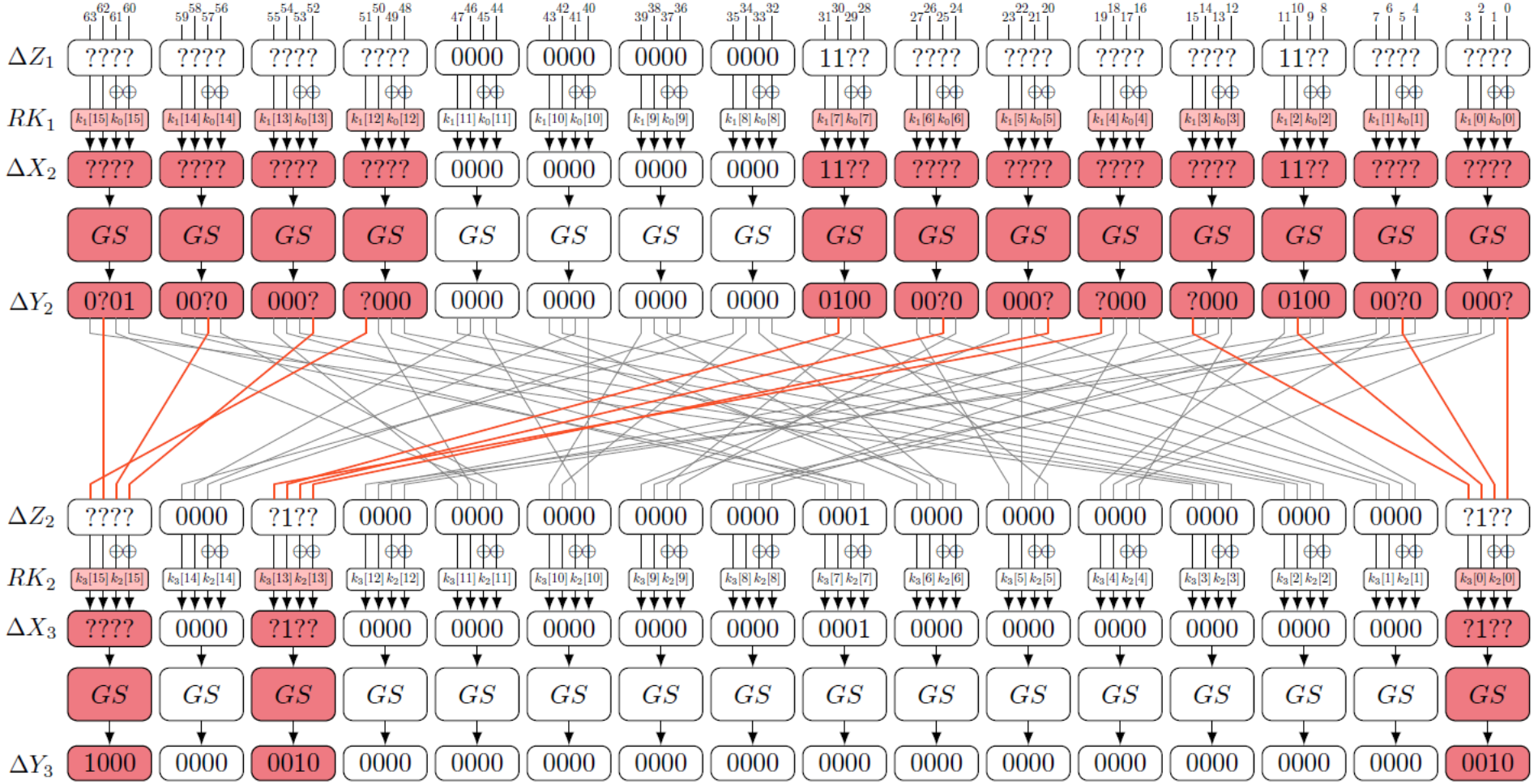
Result and extensions

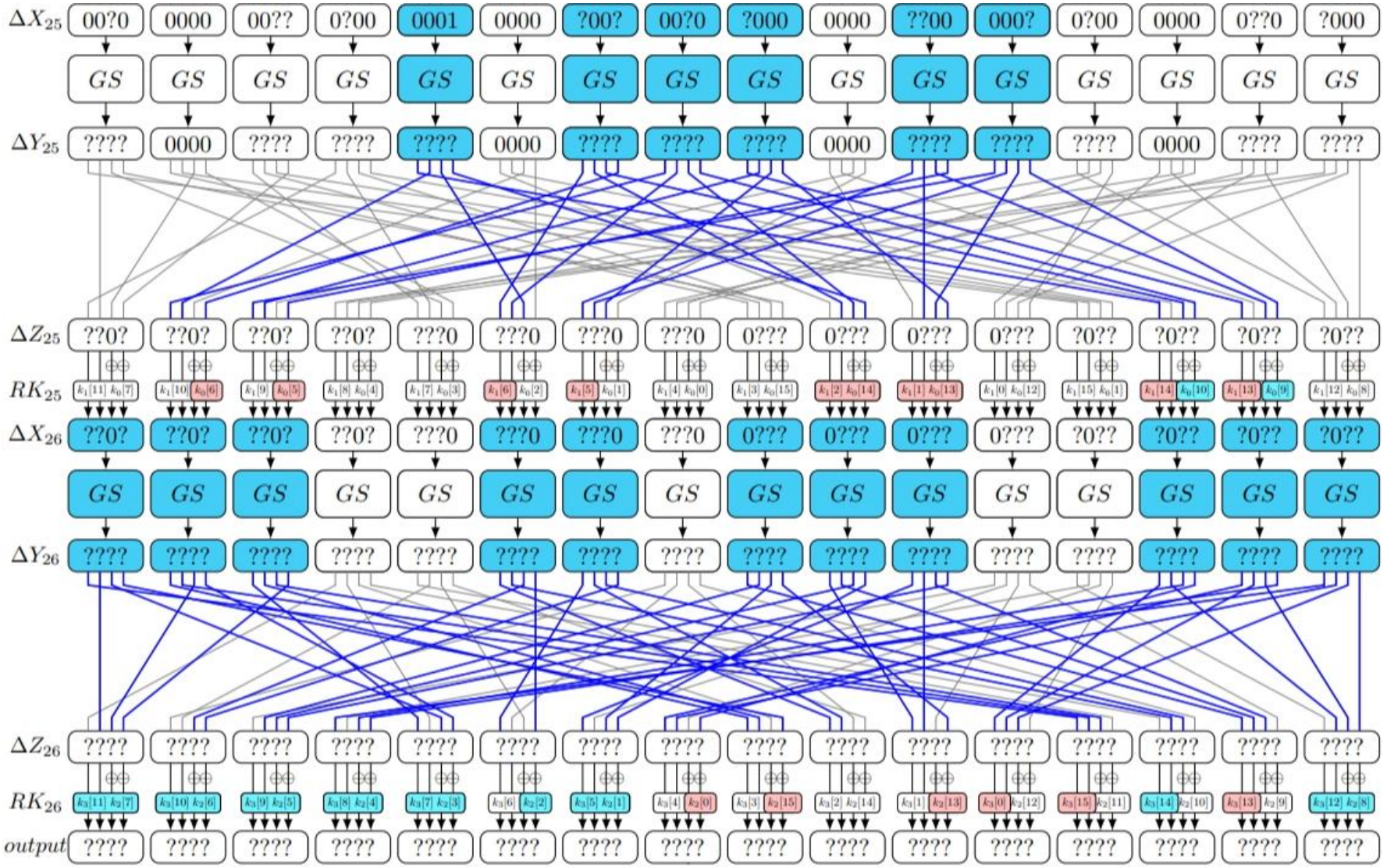
✓ Guessing strategy in E_b: Guessing m'_b=30 subkey bits to obtain r'_b=44 filtering bits.



Result and extensions

- ✓ Guessing strategy in E_f:

  Guessing m'_f=18 subkey bits to obtain r'_f=28 filtering bits *combine with reused subkey bits.*



Result and extensions

Choose $s = 2$ and $h = 20$

- ✓ E_b: $(m_b = 30,\ r_b = 44)$
  - m'_b=30, r'_b=44, r^*_b = 0
- ✓ E_f: $(m_f = 64,\ r_f = 64)$
  - m'_f=18, r'_f=28, r^*_f = 36

- $T_{total} = 2^{110.06}$ *enc.* & $2^{115.8}$ *m.a.*
- $D_{total} = 2^{63.78}$
- $M_{total} = 2^{64.36}$

$$D = y \cdot r_b$$

$$T_0 = 4 \cdot D = 2^{63.78}$$

$$T_1 = 2^{m'_b + m'_f} \cdot 4 \cdot D = 2^{111.78}$$

$$T_2 = 2^{m'_b + m'_f} \cdot 2 \cdot D \cdot 2^{r^*_b} = 2^{110.78}$$

$$T_3 = 2^{m'_b + m'_f} \cdot D^2 \cdot 2^{2r^*_b + 2r^*_f - 2n} \cdot \epsilon = 2^{115.56} \cdot \epsilon$$

$$T_4 = 2^{m'_b + m'_f + k - m'_b - m'_f - h} = 2^{k-h} = 2^{108}$$

$$D_{total} = 4 \cdot D = 2^{63.78}$$

$$\begin{aligned} M_{total} &= M_0 + M_1 + M_c \\ &= 4 \cdot D + 2 \cdot D \cdot 2^{r^*_b} + 2^{m^*_b + m^*_f} \\ &= 2^{63.78} + 2^{62.78} + 2^{36} \\ &\approx 2^{64.36} \end{aligned}$$

Result and extensions

➢ Strategy II

- ✓ E_b:
  - guessing **26** subkey bits, obtain **39** filtering bits;
- ✓ E_f:
  - guessing **22** subkey bits, obtian **34** filtering bits.

- $T_{total} = 2^{111.51}$ $enc.$ & $2^{115.78}$ $m.a.$
- $D_{total} = 2^{63.78}$
- $M_{total} = 2^{67.8}$

➢ Attack on 23-round GIFT-128

- ✓ E_b:
  - guessing **4** subkey bits, obtain **7** filtering bits;
- ✓ E_f:
  - guessing **0** subkey bits, obtian **0** filtering bits.

- $T_{total} = 2^{123.1}$ $enc.$ & $2^{126.31}$ $m.a.$
- $D_{total} = 2^{121.31}$
- $M_{total} = 2^{121.63}$

Result and extensions

Method (related-key)	Time	Data	Memory	Online
Differential	$2^{123.23}$ $enc.$	$2^{60.96}$	$2^{102.86}$	2021 ToSC
Rectangle	$2^{122.78}$ $enc.$	$2^{63.78}$	$2^{63.78}$	2022 E.C.
Rectangle	$2^{121.75}$ $enc.$	$2^{62.715}$	$2^{62.715}$	2022 Ins.C
Rectangle	$\mathbf{2^{110.06}}$ $\boldsymbol{enc.}$ & $\mathbf{2^{115.8}}$ $\boldsymbol{m.a.}$	$2^{63.78}$	$2^{64.36}$	This
Rectangle	$\mathbf{2^{111.51}}$ $\boldsymbol{enc.}$ & $\mathbf{2^{115.78}}$ $\boldsymbol{m.a.}$	$2^{63.78}$	$2^{67.8}$	This

Table. Results of 26-round attack on GIFT-64

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Summary

- The best attacks on GIFT-64 in terms of time complexity to date.
- An initial application of the generic rectangle key recovery algorithm for the bit-wise block ciphers
- The dedicated model of key recovery attack on GIFT-64.

Thank you!

Q & A

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