SMAUG: Pushing Lattice-based Key Encapsulation Mechanisms to the Limits

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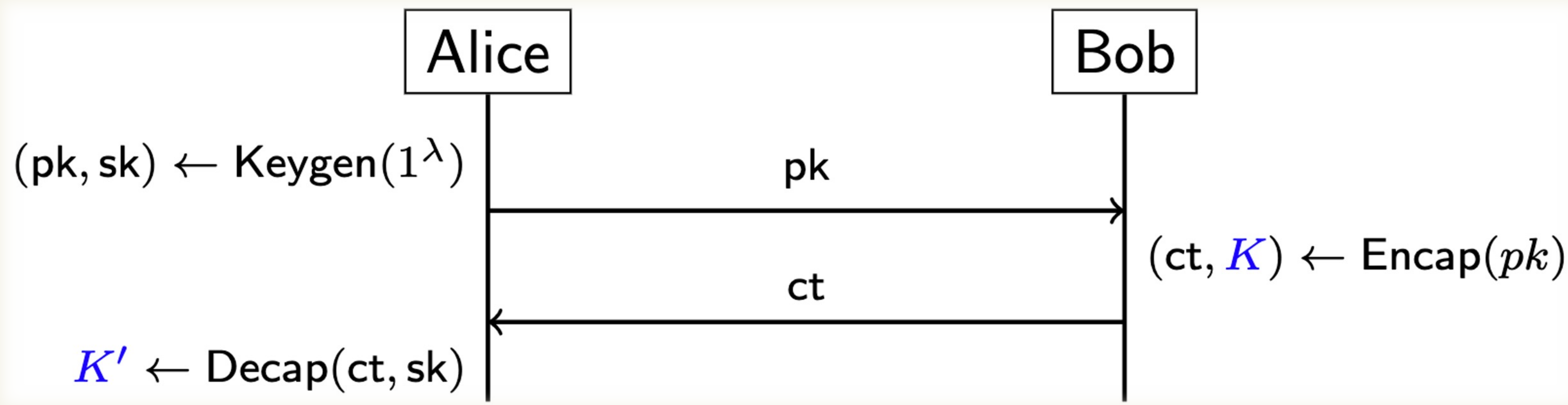
Lattice-based KEMs

KEMs in Post-Quantum World

- Key Encapsulation Mechanism (KEM)



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TLS protocols

IoT devices

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Various lattice-based KEMs:

Kyber, Saber, NTRU, Round5, FrodoKEM, Rlizard,…

Requirements for KEMs

- Efficiency

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Efficiency of Lattice-KEMs

- Recent lattice-based KEMs with low enough Decryption Failure Probability (DFP)
  - KEMs using ECCs or having higher DFPs are omitted

- FrodoKEM
- RLizard
- Kyber
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Scheme	sk	pk	ct ↑	DFP	Sec.	$\|K\|$	Assumption
Sable	800	608	672	-139	114	256	MLWR
NTRU	699	935	699	$-\infty$	106	256	NTRU
Saber	832	672	736	-120	118	256	MLWR
Kyber	1632	800	768	-139	118	256	MLWE
RLizard	385	4096	2080	-188	147	256	RLWE+RLWR
FrodoKEM	19888	9616	9752	-139	150	128	LWE

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$\Rrightarrow$ IND-CCA2 secure KEM

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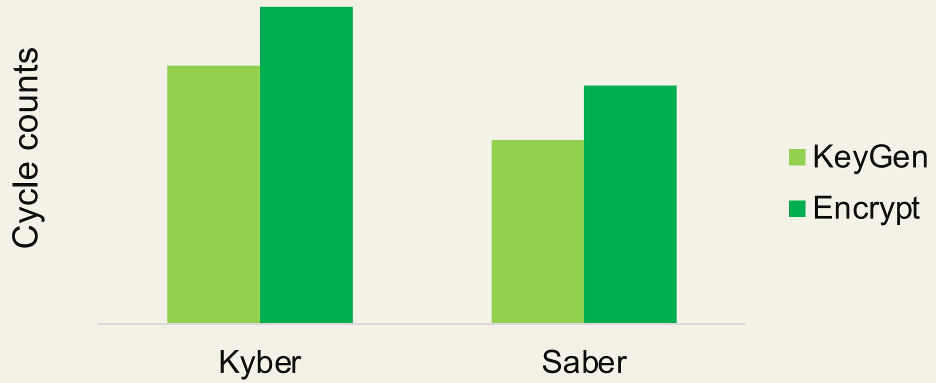

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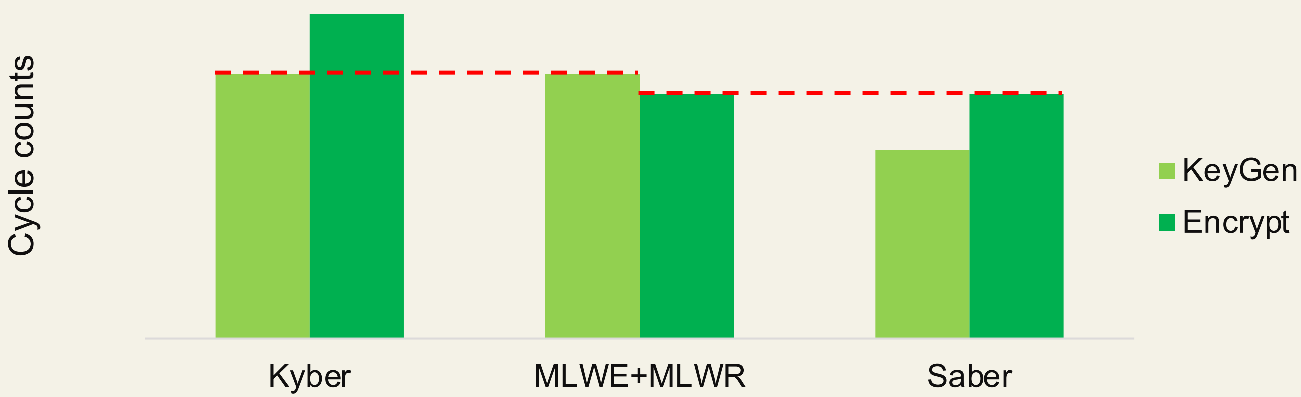

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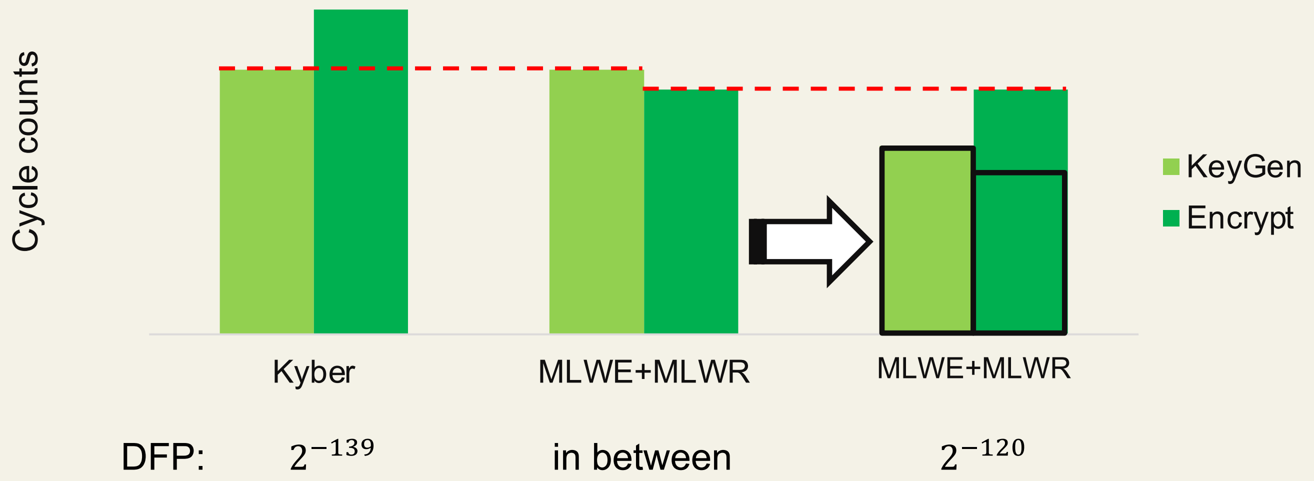

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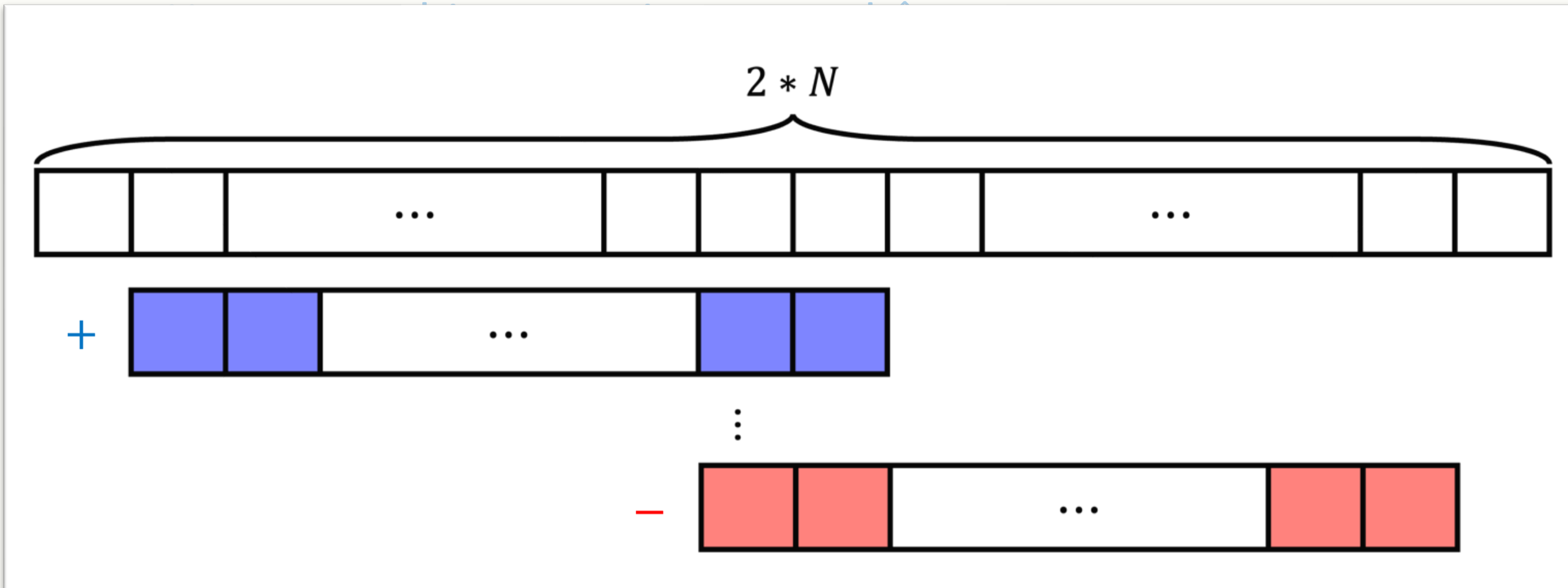

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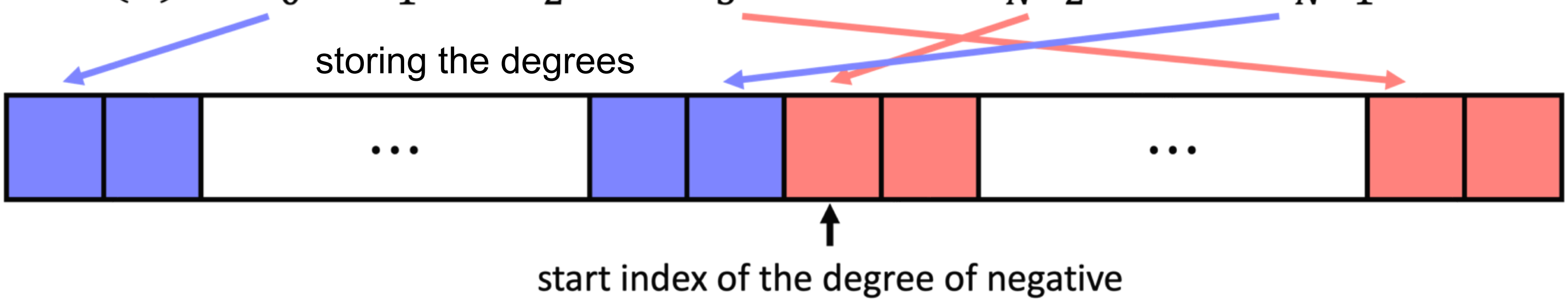
$$a(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_{N-2}x^{N-2} + a_{N-1}x^{N-1}$$



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$\Rrightarrow$ Boolean algorithm for dGaussian

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<u>dGaussian$_\sigma(x)$:</u>

Require: $x = x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9 \in \{0,1\}^{10}$

$s = s_1 s_0 = 00 \in \{0,1\}^2$

$s_0 = x_0x_1x_2x_3x_4x_5x_7\overline{x_8}$

$s_0 \mathrel{+}= (x_0x_3x_4x_5x_6x_8) + (x_1x_3x_4x_5x_6x_8) + (x_2x_3x_4x_5x_6x_8)$

$s_0 \mathrel{+}= (\overline{x_2x_3x_6}x_8) + (\overline{x_1x_3x_6}x_8)$

$s_0 \mathrel{+}= (x_6x_7\overline{x_8}) + (\overline{x_5x_6}x_8) + (\overline{x_4x_6}x_8) + (\overline{x_7}x_8)$

$s_1 = (x_1x_2x_4x_5x_7x_8) + (x_3x_4x_5x_7x_8) + (x_6x_7x_8)$

$s = (-1)^{x_9} \cdot s$ ▷ · is the arithmetic multiplication

return s

- Booleanize CDT
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⇒ Boolean algorithm for dGaussian

Implementation

Parameter Sets

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$\Rightarrow$ Smallest ciphertexts & public keys

Size Comparison

- NIST's security level 1

Schemes	Sizes (ratio)			Security	
	sk	pk	ct	Classic.	DFP
Kyber512	9.4	1.2	1.1	118	-139
LightSaber	4.8	1	1.1	118	-120
LightSable	4.6	0.9	1	114	-139
SMAUG-128	1	1	1	120	-120

- Sizes: proportion to SMAUG
- SMAUG wins, loses, tie

Full Size & Performance Comparison

- NIST's security levels 1, 3, and 5

Schemes	Sizes (ratio) sk	pk	ct	Cycles (ratio) KeyGen	Encap	Decap	Security Classic.	DFP
Kyber512	9.4	1.2	1.1	1.7	2.1	2.03	118	-139
LightSaber	4.8	1	1.1	1.21	1.58	1.44	118	-120
LightSable	4.6	0.9	1	1.1	1.48	1.39	114	-139
SMAUG-128	1	1	1	1	1	1	120	-120
Kyber768	10.4	1.1	1.1	1.38	1.84	1.75	183	-164
Saber	5.4	0.9	1.1	1.21	1.64	1.47	189	-136
Sable	5	0.8	1	1.1	1.55	1.45	185	-143
SMAUG-192	1	1	1	1	1	1	181	-136
Kyber1024	15.2	0.9	1.1	1.25	1.38	1.36	256	-174
FireSaber	8	0.7	1	1.08	1.29	1.25	260	-165
FireSable	7.8	0.7	0.9	1.03	1.25	1.22	223	-208
SMAUG-256	1	1	1	1	1	1	264	-167

- Constant-time, non-vectorized C reference codes
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*SMAUG, *The Hobbits*, J. R. R. Tolkien.