

# Quantum Cryptanalysis of OTR and OPP: Attacks on Confidentiality, and Key- Recovery

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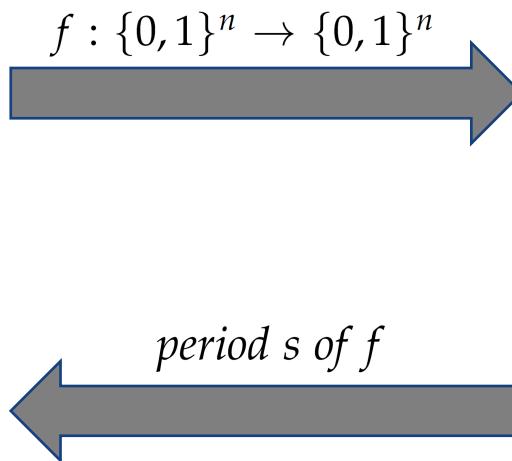
SAC'23, August 2023

# Introduction

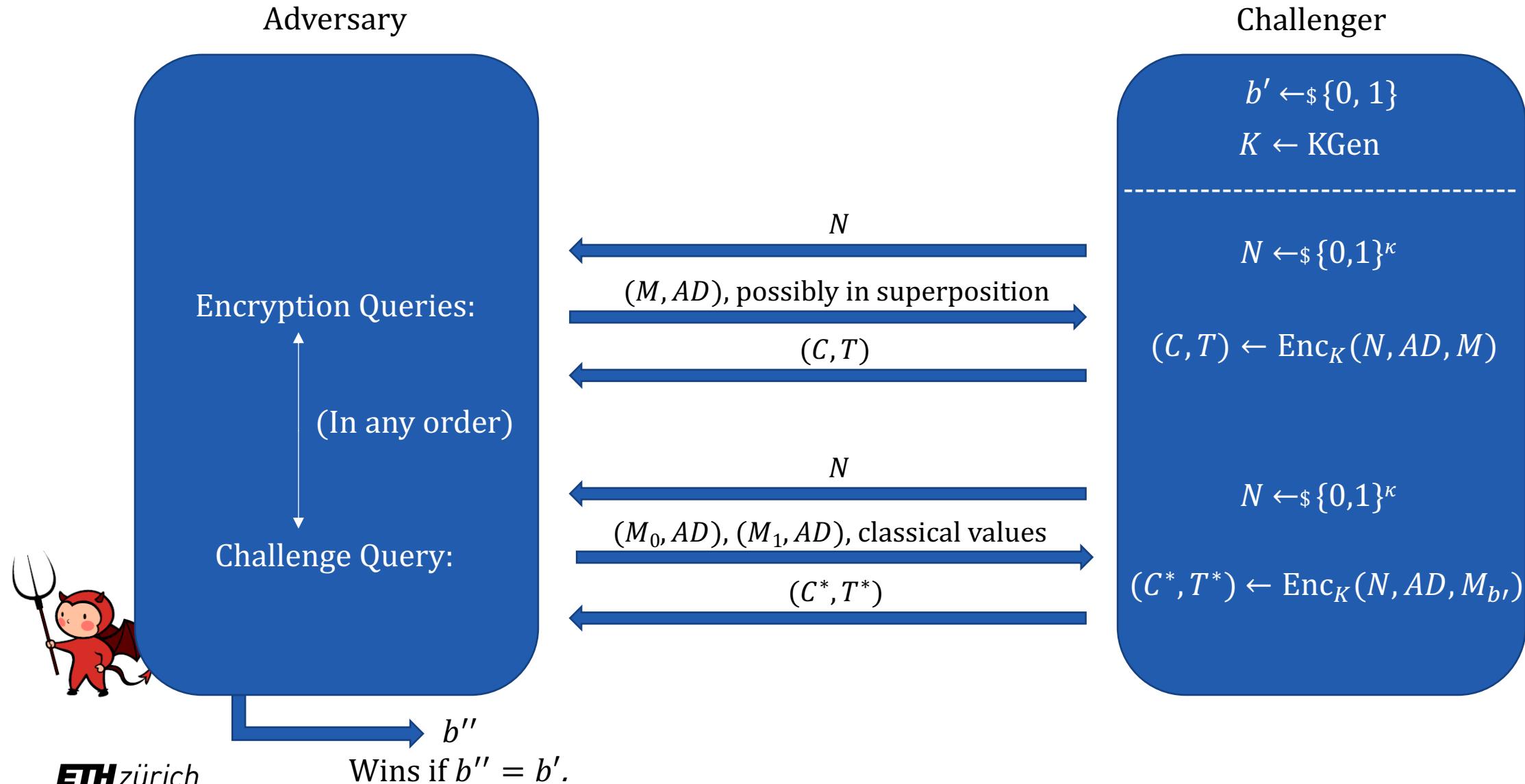
- **Quantum** security: adversary has quantum access to secret-keyed encryption or decryption oracles.
  - In contrast to **post-quantum** security: adversary has quantum access to public oracles (e.g., hash functions).
- In our setting, adversary can make encryption queries on a quantum superposition of messages.
- Prior work: quantum superposition attacks by Kaplan *et al.* (Crypto 2016) on **CBC-MAC**, **PMAC**, **GMAC**, **GCM**, **OCB**, etc. breaking unforgeability (EUF-qCMA).
- More recently: confidentiality (IND-qCPA) analysis of **OCB** modes by Maram *et al.* (ToSC 2022).
- In this work, we focus on related authenticated encryption (AE) modes **OPP** and **AES-OTR**.

# Simon's Algorithm

- Given **quantum access** to a Boolean function  $f: \{0,1\}^n \rightarrow \{0,1\}^n$  for which it holds:  
 $\exists s \in \{0,1\}^n : \forall x, y \in \{0,1\}^n$   
$$f(x) = f(y) \Leftrightarrow y \in \{x, x \oplus s\}$$
- Can recover  $s$  in  $\mathcal{O}(n)$  quantum queries (in a classical setting  $\Theta(2^{n/2})$  needed).
- In each iteration, an independent vector orthogonal to the period  $s$  is recovered with high probability.



# IND-qCPA Security Game



# Breaking AES-OTR's IND-qCPA Security

# Specifications of AES-OTR

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**Algorithm** OTR- $\mathcal{E}_{K,s}(N, A, M)$

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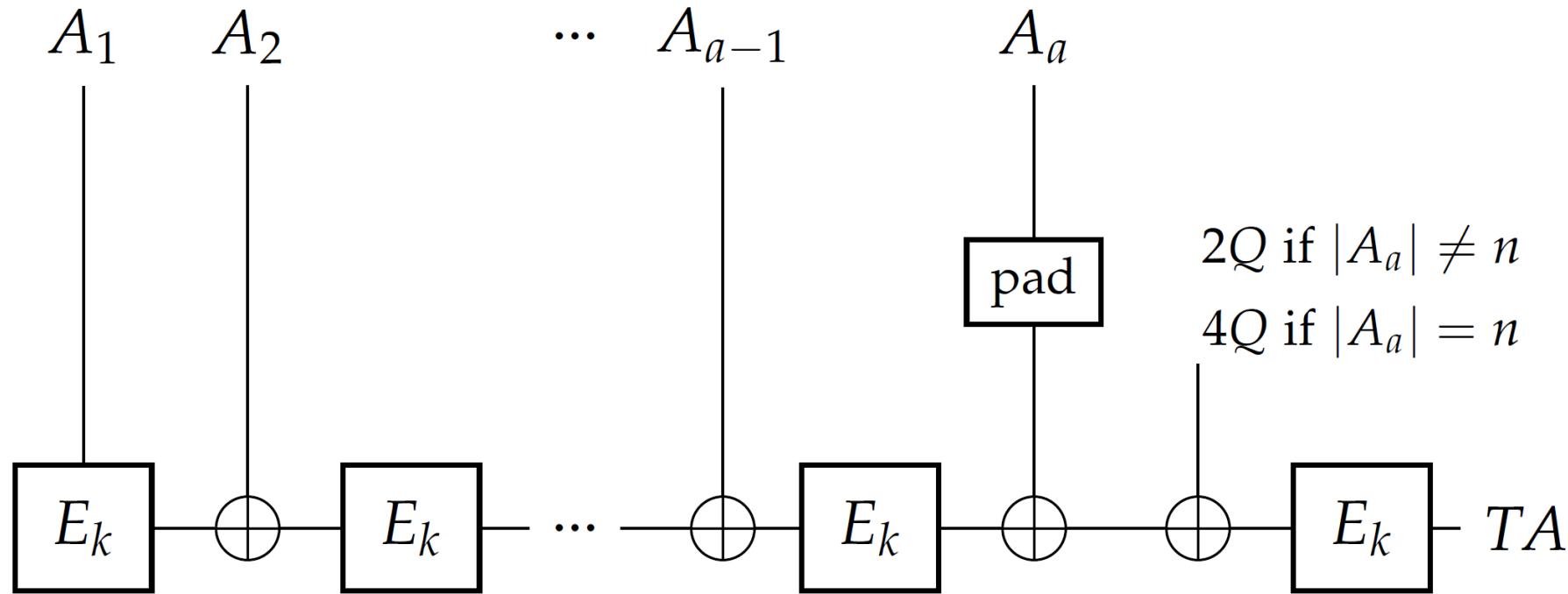
- 1: **if**  $A \neq \varepsilon$  **then**
- 2:     $TA \leftarrow \text{AF-S}_K(A)$     
- 3: **else**  $TA \leftarrow 0^n$
- 4:  $(C, TE) \leftarrow \text{EF-S}_{K,\tau}(N, M, TA)$
- 5:  $T \leftarrow \text{msb}_\tau(TE)$
- 6: **return**  $(C, T)$

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**AD Processing**

# Specifications of AES-OTR: Authentication Core $\mathbf{AF}\text{-}\mathbf{S}_K(A)$

- Associated Data  $A = A_1 \parallel \dots \parallel A_a$  processed in serial
- $Q = E_K(0^n)$



# Specifications of AES-OTR

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**Algorithm** OTR- $\mathcal{E}_{K,s}(N, A, M)$

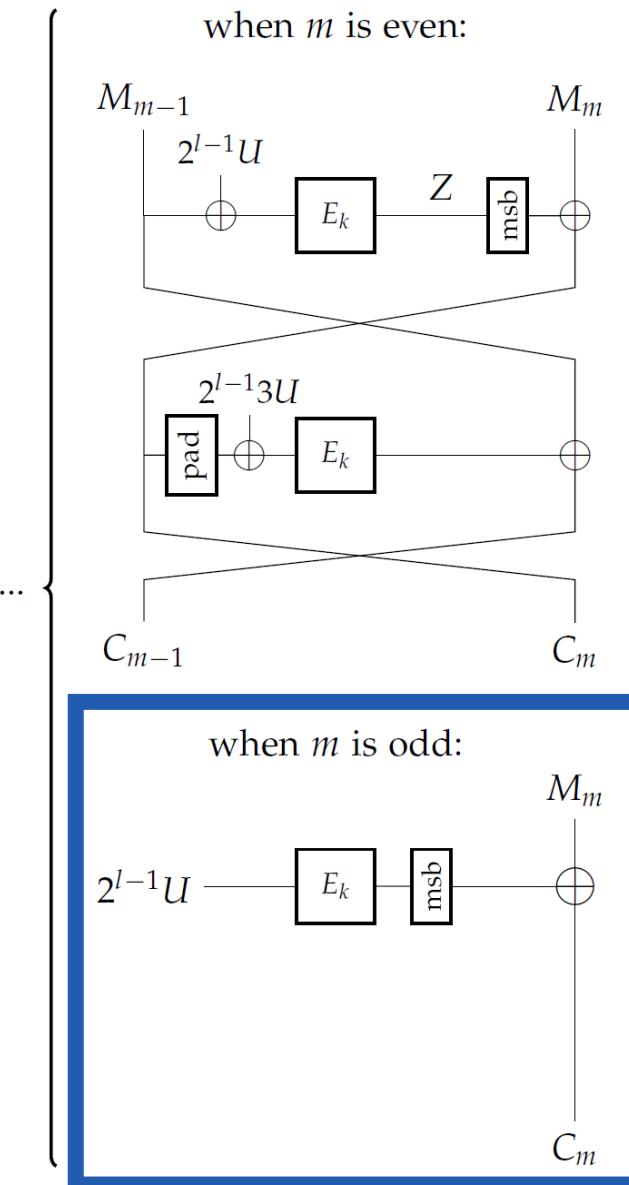
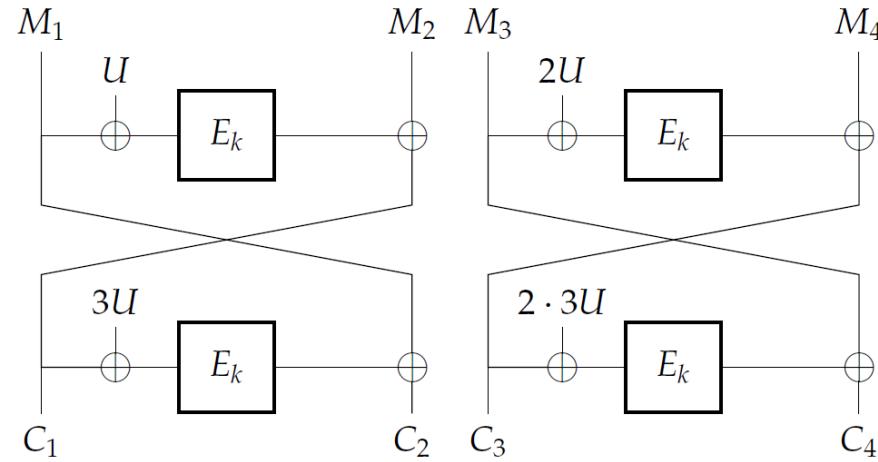
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- 1: **if**  $A \neq \varepsilon$  **then**
- 2:    $TA \leftarrow \text{AF-S}_K(A)$      
    **AD Processing**
- 3: **else**  $TA \leftarrow 0^n$
- 4:  $(C, TE) \leftarrow \text{EF-S}_{K,\tau}(N, M, TA)$      
    **Encryption Core**
- 5:  $T \leftarrow \text{msb}_\tau(TE)$
- 6: **return**  $(C, T)$

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# Specifications of AES-OTR: Encryption Core $\mathbf{EF-S}_{K,\tau}(N, M)$

- Nonce  $N$  and key  $K$
- Plaintext  $M = M_1 \parallel \dots \parallel M_m$ ,  $l = \lceil m/2 \rceil$
- $U = 2(E_K(\text{Format}(\tau, N)) \oplus TA)$
- $\text{Format}(\tau, N) = \text{bin}(\tau \bmod n, 7) \parallel 0^{n-8-\kappa} \parallel 1 \parallel N$



# Specifications of AES-OTR

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**Algorithm 2**  $\text{OTR-}\mathcal{E}_{K,s}(N, A, M)$ 

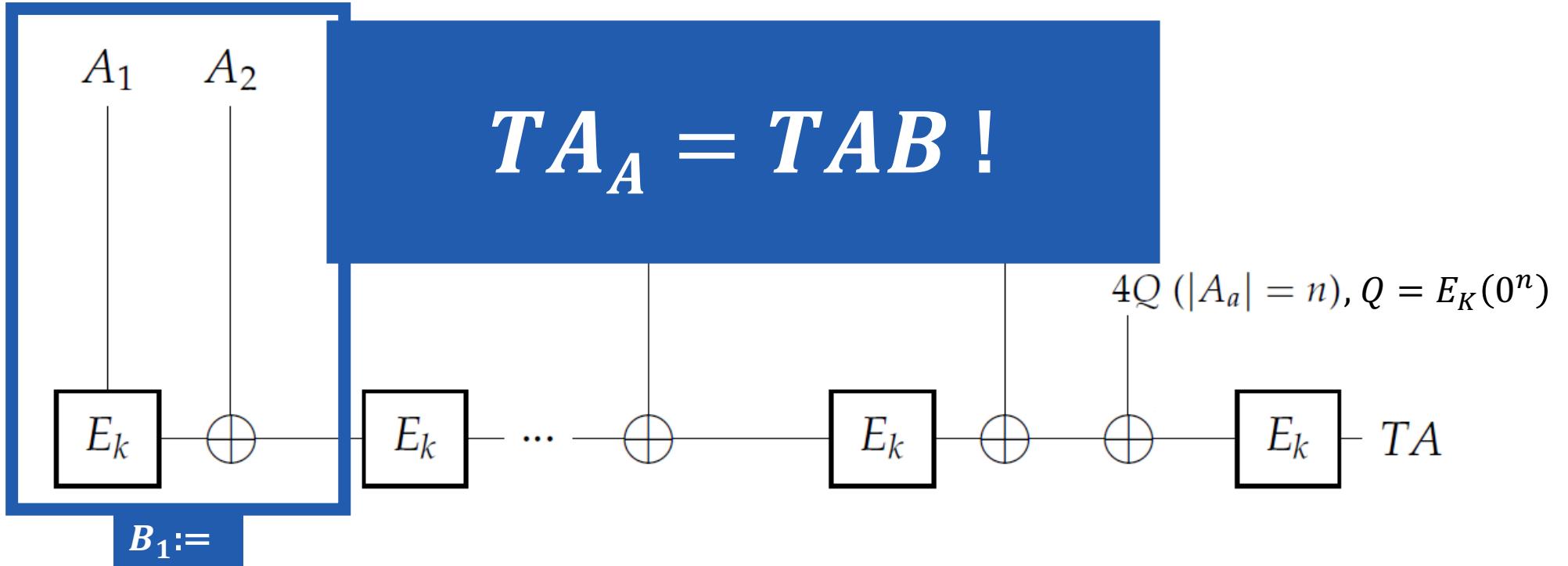
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```
1: if  $A \neq \varepsilon$  then           ← AD Processing
2:    $TA \leftarrow \text{AF-S}_K(A)$  ←
3: else  $TA \leftarrow 0^n$ 
4:  $(C, TE) \leftarrow \text{EF-S}_{K,\tau}(N, M, TA)$  ← Encryption Function
5:  $T \leftarrow \text{msb}_\tau(TE)$  ← Tag Computation
6: return  $(C, T)$ 
```

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# Finding Collisions in Serial AD Processing

- High-level attack on unforgeability first described by Kaplan *et al.* (Crypto 2016).
  - Detailed attack followed by Chang *et al.* (Symmetry 2022).
- AD  $A = A_1 \parallel \dots \parallel A_a$  with  $A_i \in \{0,1\}^n$
- Define AD  $B = B_1 \parallel \dots \parallel B_{a-1}$  with  $B_1 = A_2 \oplus E_K(A_1), B_i = A_{i+1}$



# IND-qCPA Attack on AES-OTR with Serial AD Processing

- **Raw block cipher access:** Let  $B \in \{0,1\}^n$ . Define function  $f_2 : \{0,1\}^{n+1} \rightarrow \{0,1\}^\tau$

$$f_2(b||A) = \begin{cases} \text{OTR-}\mathcal{E}_{K,s}(N, B||A, \varepsilon) & \text{if } b = 0 \\ \text{OTR-}\mathcal{E}_{K,s}(N, A, \varepsilon) & \text{if } b = 1 \end{cases}$$

with  $b \in \{0,1\}$  and  $A \in \{0,1\}^n$ .

- Where

$$\text{OTR-}\mathcal{E}_{K,s}(N, D, \varepsilon) = \text{msb}_\tau \left( E_K \left( 3^3 2 \left( TA_D \oplus E_K(\text{Format}(\tau, N)) \right) \right) \right)$$

 depends on AD

Period of  $f_2$  only depends on  $TA_D$ !

# IND-qCPA Attack on AES-OTR with Serial AD Processing

- Define new function  $g : \{0, 1\}^{n+1} \rightarrow \{0, 1\}^n$ , (inner function of  $f_2$ )

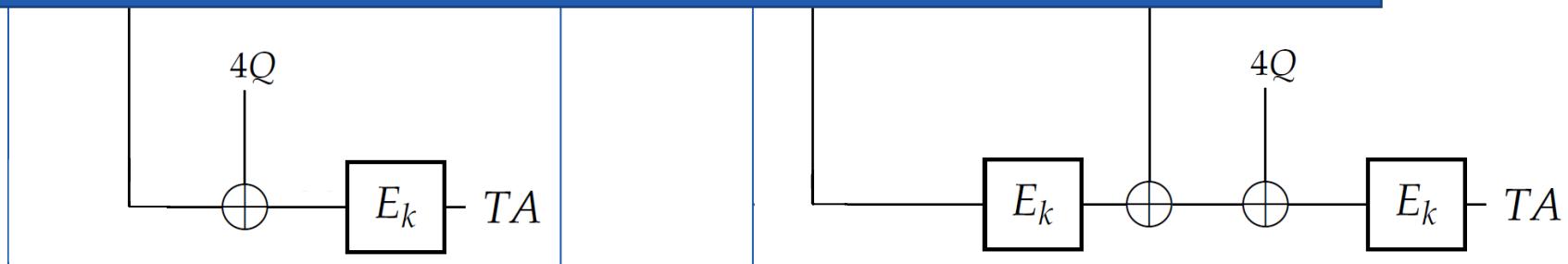
$$\begin{cases} \text{AE-S}_\nu(B \parallel A) & \text{if } h = 0 \\ \dots & \dots \end{cases}$$

- Claim:  $g$  (and the

$$g(0 \parallel A \oplus$$

We can recover  $E_k(B)$   
for any  $B \in \{0, 1\}^n$ !

$$= g(0 \parallel A)$$



# IND-qCPA Attack on AES-OTR with Serial AD Processing

Sketch of IND-qCPA attack:

1. Pick single block messages  $M_0$  and  $M_1$  and empty AD as input for the challenger.  
Record response  $(C^*, T^*)$  and the nonce  $N$ .
2. Compute

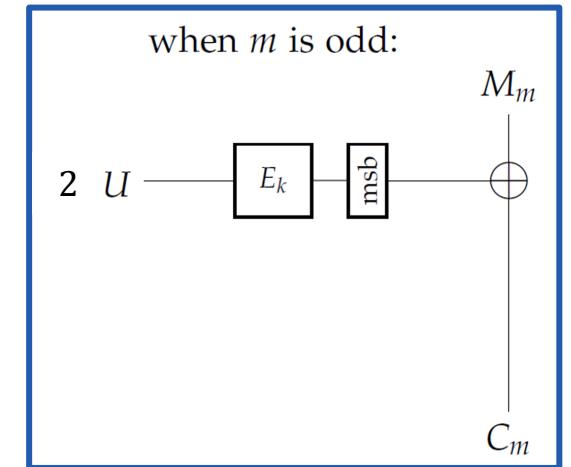
$$V = E_K \left( 2 \cdot E_K \left( \text{Format}(\tau, N) \right) \right)$$

in  $2\mathcal{O}(n)$  quantum encryption queries

3. Output the bit  $b'' = b'$  if  $M_{b'} = C^* \oplus V$

Why does this work?

→ For empty AD and single block message:

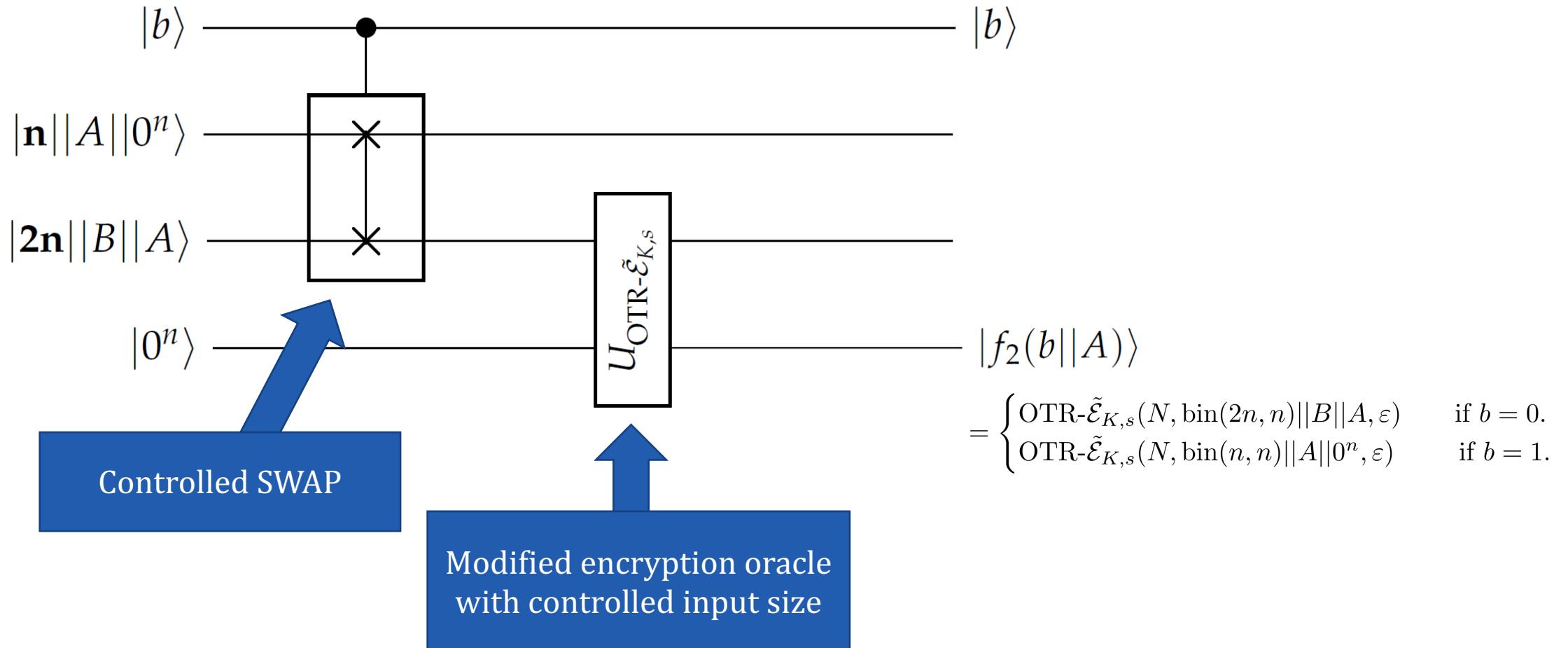


$$U = 2(E_K(\text{Format}(\tau, N)) \oplus TA)$$

$$\text{OTR-}\mathcal{E}_{K,s}(N, \varepsilon, M) \Big|_C = E_K \left( 2 \cdot E_K \left( \text{Format}(\tau, N) \right) \right) \oplus M$$

# Superposition Over Unequal-Length Data

- We need to have quantum access to  $f_2$  with a single query to the encryption oracle!

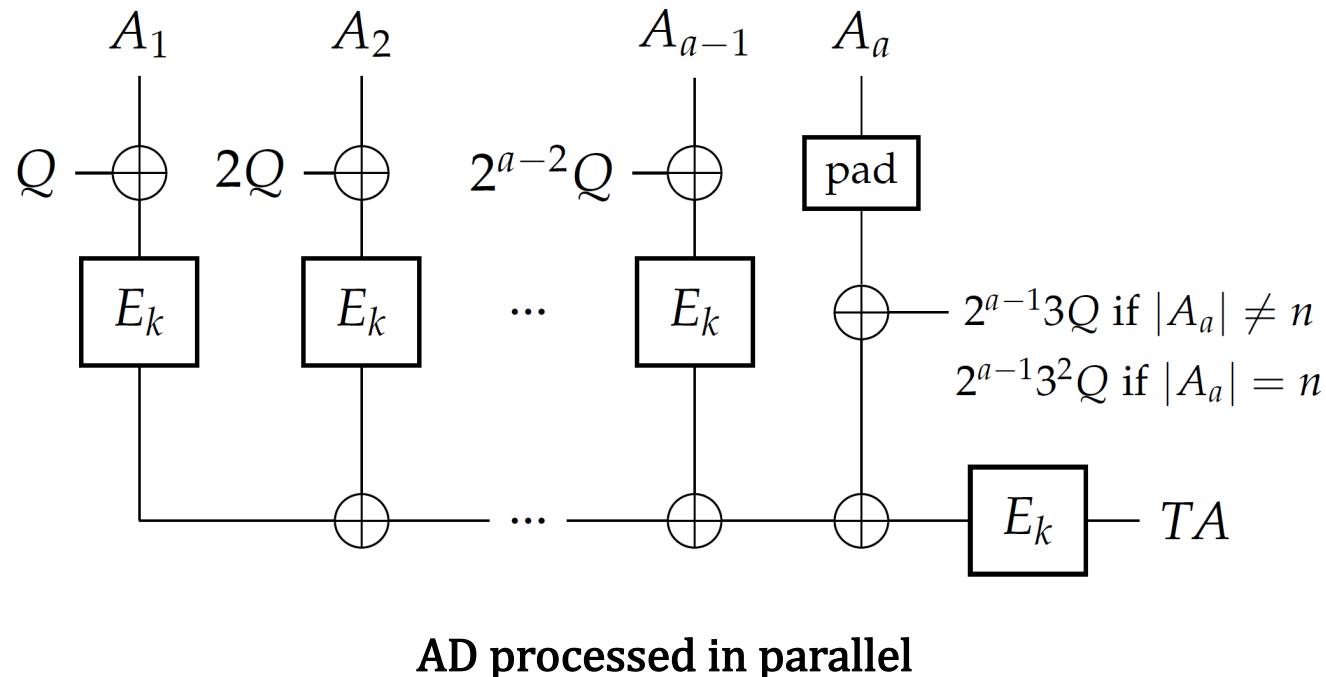


# Superposition Over Unequal-Length Data

- Laws of quantum physics define a superposition only over states with the same number of qubits.
- We can overcome this restriction in the IND-qCPA setting with this modified quantum encryption oracle!
- Allows for **stronger** quantum attacks: e.g., we gain raw block cipher access directly via Simon's algorithm.
  - This contrasts with the IND-qCPA attacks against OCB by Maram *et al.* (ToSC 2022) which also requires Deutsch's algorithm, along with Simon's algorithm.
- This model can also be extended to cryptanalysis in the more realistic **post-quantum** setting – e.g., attacking (public) hash functions.

## Further Attacks on AES-OTR

- IND-qCPA attack when AD is processed in parallel.
- IND-qCPA attack when AD is always empty.



# Quantum Key-Recovery Attack on OPP

# Specifications of OPP (simplified)

- Offset Public Permutation Mode

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**Algorithm**     $\text{OPP-}\mathcal{E}(K, N, AD, M)$ 

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- 1:  $X \leftarrow \text{pad}_{n-\kappa-k}^0(N)$   Zero padding
- 2:  $C, S \leftarrow \text{OPPEnc}(K, X, M)$   Encryption Core
- 3:  $T \leftarrow \text{OPPAbs}(K, X, AD, S)$   Authentication Core  
(not relevant for the attack)
- 4: **return**  $C, T$

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# Specifications of OPP: Encryption Core $\mathbf{OPP}\mathbf{Enc}(K, X, M)$

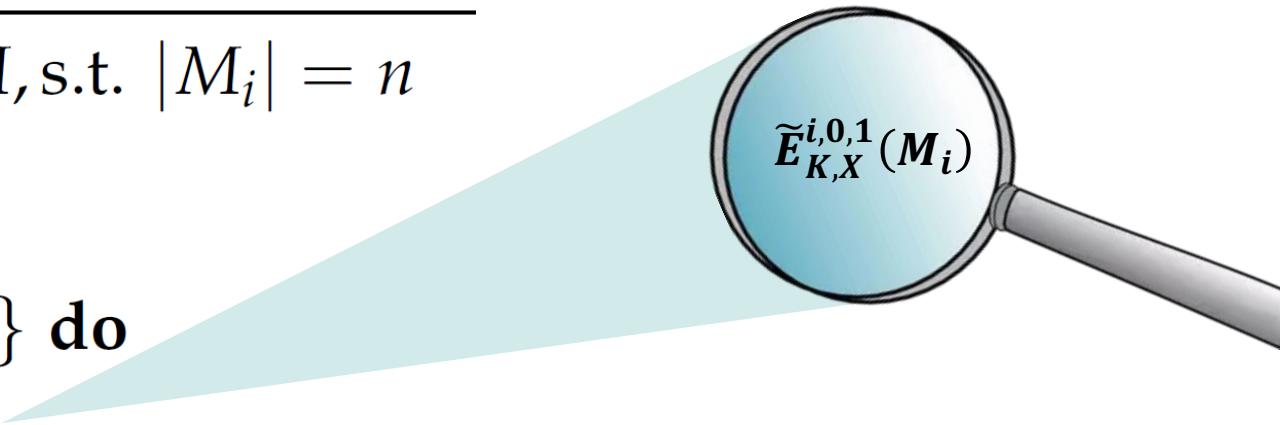
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## Algorithm $\mathbf{OPP}\mathbf{Enc}(K, X, M)$

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```
1:  $M_0||\dots||M_{m-1} \leftarrow M$ , s.t.  $|M_i| = n$ 
2:  $C \leftarrow \varepsilon$ 
3:  $S \leftarrow 0^n$ 
4: for  $i \in \{0, \dots, m-1\}$  do
5:    $C_i \leftarrow \tilde{E}_{K,X}^{i,0,1}(M_i)$ 
6:    $C \leftarrow C||C_i$ 
7:    $S \leftarrow S \oplus M_i$ 
8: return  $C, \tilde{E}_{K,X}^{m-1,2,1}(S)$ 
```

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Checksum: Xor of all plaintext blocks

What is  $\tilde{E}_{K,X}^{i,0,1}(M_i)$ ?

$$X = \text{pad}_{n-\kappa-k}^0(N)$$

$$\bar{i} = (i_0, i_1, i_2) \in \mathbb{N}^3$$

Key

$$\tilde{E}(K, X, \bar{i}, M) = P\left(\delta(K, X, \bar{i}) \oplus M\right) \oplus \delta(K, X, \bar{i})$$

$$\gamma^{i_2} \circ \beta^{i_1} \circ \alpha^{i_0}(P(X \parallel K))$$

$$\gamma(x) = \varphi^2(x) \oplus \varphi(x) \oplus x$$

$$\beta(x) = \varphi(x) \oplus x$$

$$\alpha(x) = \varphi(x)$$

$$\varphi(x) = Mx, M \text{ a matrix}$$

$$\delta(K, X, (i, 0, 1)) = \varphi^{i+2}(\Omega) \oplus \varphi^{i+1}(\Omega) \oplus \varphi^i(\Omega) \quad \Omega = P(X||K)$$

# Quantum Key-Recovery Attack on OPP: Preparation

- Ciphertext block as a function of its corresponding plaintext block:  $f_i : \{0,1\}^n \rightarrow \{0,1\}^n$

What if we can recover

$$\Omega = P(X||K)?$$

# Quantum Key-Recovery Attack on OPP

- **Idea:** (By Bhaumik *et al.* (Asiacrypt 2021)) Create periodic function that contains  $n$  copies of the earlier periodic function in the linear function  $g : \{0,1\}^{(2n+1)n+\tau} \rightarrow \{0,1\}^{(n+1)n}$

$$g(C_0, C_1, \dots, C_{2n}, t) = (C_0, C_1 \oplus C_2, \dots, C_{2n-1} \oplus C_{2n})$$

- Using  $g$ , define  $\tilde{f}_N : \{0,1\}^{n^2} \rightarrow \{0,1\}^{(n+1)n}$  such that

$$\tilde{f}_N(M_1, \dots, M_n) = g \circ \text{OPP-}\mathcal{E}(K, N, \varepsilon, 0^n || M_1 || M_1 || M_2 || \dots || M_n || M_n)$$

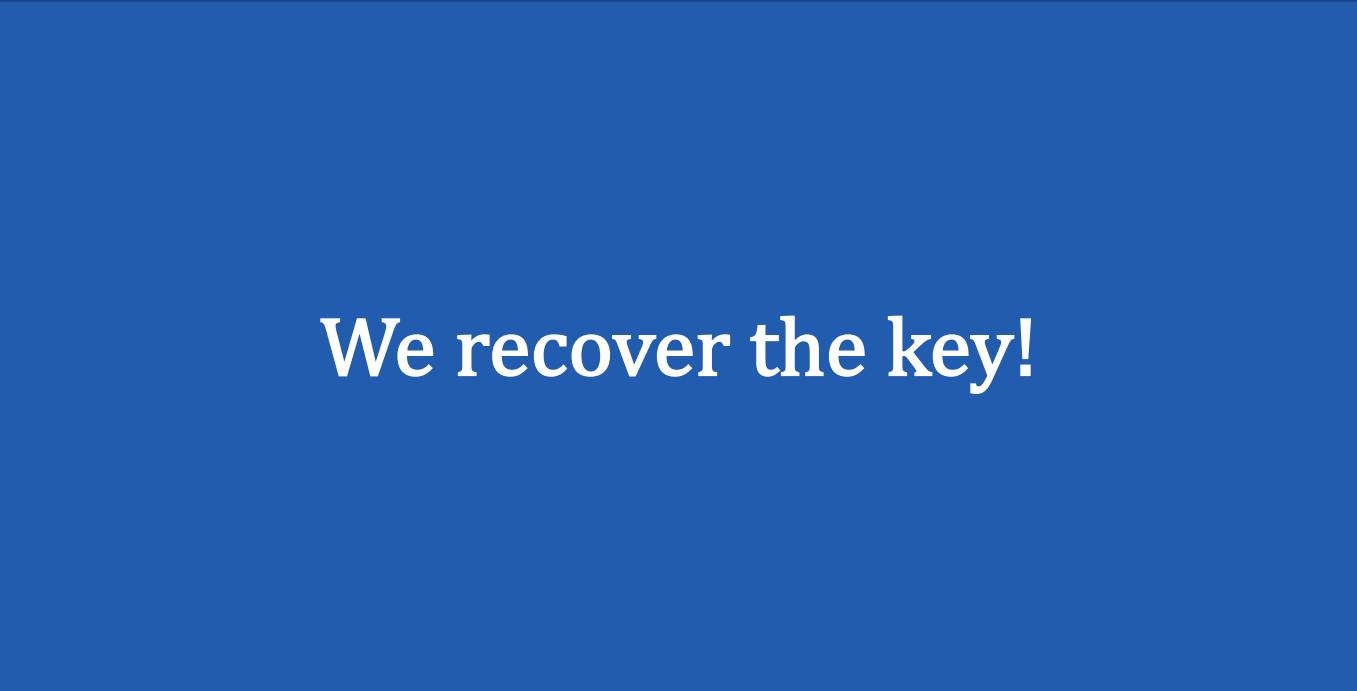
- $\tilde{f}_N$  has  $n$  linearly independent periods  $\langle s_i \rangle_{i \in [n]}$

$$s_i = \left( (0^n)^{i-1} || \varphi^{2i+2}(\Omega) \oplus \varphi^{2i-1}(\Omega) || (0^n)^{n-i} \right)$$

$$\Omega = P(X || K)$$

# Quantum Key-Recovery Attack on OPP

- Apply Simon's algorithm and recover  $y = (y_1, \dots, y_n) \in \{0, 1\}^{n^2}$  orthogonal to **each** of the periods with a single quantum query
- We get  $n$  linear equ



We recover the key!

which we are able to

- $P$  is a public, efficient



For questions, please reach out to the authors:

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# Extra Slide

- Periodicity of  $f_2$ :

$$\begin{aligned} g(1||A \oplus 1||E_K(B)) &= g(0||A \oplus E_K(B)) = \text{AF-}S_K(B||A \oplus E_K(B)) \\ &= E_K(4Q \oplus A \oplus E_K(B) \oplus E_K(B)) = E_K(4Q \oplus A) \\ &= \text{AF-}S_K(A) = g(1||A) \end{aligned} \tag{3.4.2}$$

- $f_2$  is not periodic when computed with two quantum encryption queries:

$$\begin{aligned} f_2(0||A \oplus 1||E_K(B)) &= f_2(1||A \oplus E_K(B)) \stackrel{\text{def}}{=} \text{OTR-}E_{K,s}(N_2, A \oplus E_K(B), \varepsilon) \\ &\neq \text{OTR-}E_{K,s}(N_1, B||A, \varepsilon) \stackrel{\text{def}}{=} f_2(0||A) \end{aligned}$$